$\qquad$

## Probability

## Terms, Postulates and Theorems

Sample Space - All possible outcomes

- 52 cards in a deck


Universal Sample Space - the set containing all objects or elements and of which all other sets are subsets

- Find the sample space of flipping a coin twice

Subset - a set of which all the elements are contained in another set

- Find the sample space of flipping a coin twice and getting at least one head
$\subset=$ subset smaller
$\subseteq=$ subset smaller or equal to

Fundamental Counting Principle - If there are $x$ ways for a first event to happen and $y$ ways for a second event to happen, then there are $x y$ ways for both to occur.

- Find how many dinner combinations that can be found by ordering 1 main dish, 1 side dish, and 1 beverage if you can choose between 10 main dishes, 6 side dishes and 3 beverages.

Probability - The likelihood of an event happening.

$$
\operatorname{Prob}(\text { event })=\frac{\# \text { of favorable outcomes }}{\# \text { total outcomes }}
$$

The Complement of an Event, "not" (Every element that is not in the event)

- Find the probability of landing on yellow.

- Find the probability of not landing on yellow.

Uniform Probability - Each element in the sample space is equally likely to happen

- Example

Non-Uniform Probability - Each element in the sample space is not likely to happen

- Example

Sample Space using Venn diagrams

- \# of Boys, \# of Girls, \# of Xbots

Sample Space using Tree Diagrams

- Flipping a coin 2 times

Sample Space using Two-Way Table

- Flipping a coin 2 times


## Mutually Exclusive or Disjoint Sets

Sets that have no element in common

## The Intersection, "AND" $\cap$

The elements sets have in common


The Intersection

The Union, "OR" U
The combination of all elements in sets

## Mutually Exclusive $=$ = Independence

## Conditional Probability

A probability that event $A$ happens given that event $B$ has already happened.

- Find the probability that Jim skips school given that it is a Tuesday.

Prob(Skip|Tuesday)

Independent Events (Simple)
Independent Events - When the first event doesn't influence the chance of the second event. Key word: REPLACEMENT

1. $\quad P(A \cap B)=P(A) \cdot P(B)$

Find the probability that your card is black AND a king.
2. $\quad P(A \cup B)=P(A)+P(B)-P(A \cap B)$

Find the probability that your card is black $O R$ a king.
3. $\quad P(A \mid B)=P(A)$

Given that you just flipped a coin heads, what is the probability the next card you get is a king?

## Dependent Events (Complicated)

Dependent Events - When the first event influences the chance of the second event. Key word: NON-REPLACEMENT

1. $P(A \cap B)=P(A) \cdot P(B \mid A)$

- $P(A \cap B) \neq P(A) \cdot P(B)$

Find the probability that your first card is black AND your second card is red.
2. $\quad P(A \cup B)=P(A)+P(B)-P(A \cap B)$

$$
-P(A \cap B) \neq P(A) \cdot P(B)
$$

Find the probability that your first card is black OR your second card is red.
3. $\quad P(A \mid B) \neq P(A)$

Given that you just picked a black king, what is the probability the next card you get is black?

## Venn Diagrams

## House Party

You are having dinner guests with 12 of your closest friends. As the host you serve lasagna and pumpkin pie. Nine of your friends eat lasagna, 6 of your friends eat pumpkin pie and 2 ate nothing

P.1: Show the sample space of this dinner party.
P.1: If you choose a friend at random, what is the chance the friend ate lasagna only?
P.3: Find the probability of choosing a person that ate lasagna given that they ate pie.
P.3: Find the chance of choosing someone who ate pie given that the person ate lasagna.
P.1: If you choose a student at random, what is the chance the student rides the bus to school and plays a sport for the school?
P.3: Find the chances of choosing a person who plays a sport under the condition that the person rides the bus.
P.3: Find the chances of choosing a person who rides the bus under the condition that the person plays a sport.

## Tree Diagram

## Coinage 1

You flip a coin.
Day 2: List the sample space and probabilities of each outcome.

## Coinage 2

You flip a coin twice.

Day 2: List the sample space and probabilities of each outcome.

## Coinage 3

You flip a coin thrice.

Day 2: List the sample space and probabilities of each outcome.

## Notes Section P. 2

## Roll That Die

Deuce is obsessed with 2 s . He's so obsessed that he rolls a die 2 times. If Deuce is only concerned with getting a 2 on each die, list the sample space of getting and not getting 2 s . Find the probability of each outcome.

Flip a coin save the die
Flip a coin and roll a die. What is the probability that you flipped a head and rolled a composite number?
$\qquad$

## Two-Way Frequency Tables

Two-way frequency tables help to organize data.

|  | iPod | NO iPod | Total |
| :---: | :---: | :---: | :---: |
| Smart <br> Phone | 20 | 18 | 38 |
| NO Smart |  |  |  |
| Phone |  |  |  |
| Total | 8 | 4 | 12 |
|  | 28 | 22 | 50 |
|  |  |  |  |

Determine the information from the two-way frequency table.

|  | Sports |  | Video | Dance |
| :---: | :---: | :---: | :---: | :---: |
| Total |  |  |  |  |
|  | 13 | 16 | 1 | 30 |
| Boys | 13 | 5 | 15 | 30 |
| Total | 10 | 5 | 21 |  |
|  | 23 | 21 | 16 | 60 |
|  |  |  |  |  |

1. How many students in the class?
2. How many girls like to dance?
3. How many students like to play sports?
4. How many girls don't like to play video games?

Notes Section P. 4
Complete the two-way frequency table that represents the given information.
5. 15 and 30 -year-old males were asked which of the following actors they liked the best as Batman and the following results were found.

6. $P($ Adam West $)=$
7. $P(15$ yr old $)=$
8. $P($ Christian Bale $)=$
9. $P(30$ yr old $\cap$ George Clooney $)=$
10. $P(15$ yr old $\cap$ NOT Christian Bale $)=$

Complete the table from the given information.
11. 23 Juniors and 31 Seniors were asked about which class they like better between AP World History and AP Calculus. 41 students picked AP Calculus and 11 juniors picked AP World History.

"And" is the intersection of a column and row.

|  | iPod | NO iPod | Total |
| :---: | :---: | :---: | :---: |
| Smart | 20 | 18 | 38 |
| Phone | 20 | 4 | 12 |
| NO Smart <br> Phone | 8 | 22 | 50 |
|  |  |  |  |

1. $P($ Smart Phone $\cap i P o d)=$
2. $P($ No Smart Phone $\cap$ No iPod $)=$
3. $P($ No Smart Phone $\cap i P o d)=$
"Or" is the sum of a row and column minus the intersection.

|  | iPod | NO iPod | Total |
| :---: | :---: | :---: | :---: |
| Smart <br> Phone | 20 | 18 | 38 |
| NO Smart |  |  |  |
| Phone |  |  |  |
|  | 8 | 4 | 12 |
|  | 28 | 22 | 50 |
|  |  |  |  |

4. $P($ Smart Phone $\cup$ iPod $)=$
5. $P($ No Smart Phone $\cup$ No iPod $)=$

## Conditional Probabilities in Two Way Frequency Tables

Given that something occurs, restricts the sample space to a row or column.

|  | iPod | NO iPod | Total |
| :---: | :---: | :---: | :---: |
| Smart <br> Phone | 20 | 18 | 38 |
| NO Smart |  |  |  |
| Phone |  |  |  |
| Total | 8 | 4 | 12 |
|  | 28 | 22 | 50 |
|  |  |  |  |

6. $P($ Smart Phone $\mid$ iPod $)=$
7. $P($ No Smart Phone $\mid$ iPod $)=$
8. $P($ iPod $\mid$ No Smart Phone $)=$
9. $P($ No iPod $\mid$ No Smart Phone $)=$

Determining Independence in Two-Way Tables
Remember there are two tests for independence that we know of:

TEST \#1 - If $P(A \cap B)=P(A) \cdot P(B)$

TEST \#2 - If $P(A \mid B)=P(A)$

|  | iPod | NO iPod | Total |
| :---: | :---: | :---: | :---: |
| Smart <br> Phone | 20 | 18 | 38 |
| NO Smart |  |  |  |
| Phone |  |  |  |
| Total | 8 | 4 | 12 |
|  | 28 | 22 | 50 |
|  |  |  |  |

10. Determine Independence

TEST \#1 - If $P(i \cap S P)=P(i) \cdot P(S P)$, then INDY

TEST \#2 - If $P(i \mid S P)=P(i)$, then INDY

## Two Way Relative Frequency Tables

Two Way Frequency Table
Red Green Blue Yellow Total

| Male | $\mathbf{1 2}$ | $\mathbf{7}$ | $\mathbf{9}$ | $\mathbf{0}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{2 8}$ |  |  |  |  |
| Female | $\mathbf{8}$ | $\mathbf{8}$ | $\mathbf{3}$ | $\mathbf{3}$ |
| $\mathbf{2 2}$ |  |  |  |  |
|  | Total | $\mathbf{2 0}$ | $\mathbf{1 5}$ | $\mathbf{1 2}$ |
|  | $\mathbf{3 0}$ |  |  |  |


$P($ Male $)=$
$P($ Red $)=$
$P($ Yellow $)=$
$P($ Male $\cap$ Red $)=$
$P($ Male $\cup$ Red $)=$

## Chapter Probability Summary

1. Summarize the main idea of the chapter
2. Terms (Include name and definition). Also include key example or picture for each term
3. Theorems and Postulates. Also include key example for each theorem or postulate
4. Key examples of the most unique or most difficult problems from notes, homework or application.

## Perimeter

## Notes Section 14.1

POLYGON PERIMETER
Perimeter refers to the distance around the edge of a closed figured shape. Perimeter formulas are often quite simple because they sum of the sides of a polygon.


CIRCLE PERIMETER - CIRCUMFERENCE

The perimeter of a circle is called its circumference.

Circumference $=\pi \mathrm{d}=\mathbf{2 \pi r}$

Find the missing measurement.

1. $C=9 \pi \mathrm{~cm}$, find $r$.
2. $r=16 \mathrm{~cm}$, find $d$.
3. $r=3 \mathrm{~cm}$, find $C$ with exact answer.
4. $C=\pi \mathrm{cm}$, find d .

Determine the perimeter of the given shapes. (Lines that appear to be perpendicular are perpendicular.)
5.

6.

7.


## PARALLELOGRAMS: <br> $\mathbf{A}=($ base $)($ height $)$



TRY IT! Find the area of the following:


(3)

TRIANGLES:

$$
\mathbf{A}=\frac{1}{2}(\text { base })\left(h_{\text {eight }} \text { ) }\right)
$$

Altitude $=$

TRY IT: Find the area of the following:
(4)


| SPECIAL RIGHT | PYHTAGOREAN | TRIG FUNCTIONS |
| :---: | :---: | :---: |
| TRIANGLES | THEOREM | $\sin$ |
| $\mathbf{3 0}^{\circ}-60^{\circ}-90^{\circ}$ | $\boldsymbol{a}^{2}+\boldsymbol{b}^{2}=\boldsymbol{c}^{2}$ | $\cos$ |
| $45^{\circ}-\mathbf{4 5}^{\circ}-\mathbf{9 0}^{\circ}$ |  | $\tan$ |

Find the area of the following:
$\rightarrow$



Bring the pain


Area $=16 \mathrm{~cm}^{2}$

## TRAPEZOIDS: <br> $\mathbf{A}=\frac{1}{2}\left(\right.$ bax, + bax $x_{2}$ ) height



Find the area of the trapezoid:
(1)


Find the height of the trapezoid:
(2)


KITES AND RHOMBUSES:
$\mathbf{A}=\frac{1}{2}$ deagonal $^{\text {diagonal }} \mathrm{I}_{2}$


$$
\text { Parallelogram }=\quad \text { Trapezoid }=
$$



TRY IT: Find the area of the following:


## PYTHAGOREAN THEOREM


(1)

Kite


Area of Regular Polygons Notes Section 14.4
Regular Polygons: All Sides and angles are congruent.


$$
m \angle C \text { entral }=\frac{360^{\circ}}{n}
$$

Area of a Regular Polygon $=\frac{1}{2}$ (apothem) (perimeter)


Finding the central angle:
(2)

(3)

(4)


| SPECIAL RIGHT | PYHTAGOREAN | TRIG FUNCTIONS |
| :---: | :---: | :---: |
| TRIANGLES | THEOREM | sin |
| $\mathbf{3 0}^{\circ}-60^{\circ}-90^{\circ}$ | $a^{2}+b^{2}=\boldsymbol{c}^{\mathbf{2}}$ | $\cos$ |
| $45^{\circ}-45^{\circ}-\mathbf{9 0}^{\circ}$ |  | $\tan$ |

## TRY IT: Find the area of the following regular polygons:

Octagon

| Apothem $=4 \mathrm{ft}$ |
| :--- |
| Side $=6 \mathrm{ft}$ |



POLYGONS

| \# of Sides | NAME |
| :---: | :---: |
| 3 | Triangle |
| 4 | Quadrilateral |
| 5 | Pentagon |
| 6 | Hexagon |
| 7 | Heptagon |
| 8 | Octagon |
| 9 | Nonagon |
| 10 | Decagon |
| 11 | Undecagon |
| 12 | Dodecagon |
| n | n-gon |

Circles \& Arcs
AREA OF A CIRCLE
$\mathrm{A}_{\text {circle }}=\pi \mathrm{r}^{2}$

AREA OF A CIRCLE SECTOR


AREA $_{\text {CIRCLI SECTOR }}=\frac{x^{\circ}}{360} \pi r^{2}$

Notes Section 14.5
Find the area of each region. Give exact answers.
1.

2.


4.


## Chapter 14 Summary

1. Summarize the main idea of the chapter
2. Terms (Include name and definition). Also include key example or picture for each term
3. Theorems and Postulates. Also include key example for each theorem or postulate
4. Key examples of the most unique or most difficult problems from notes, homework or application.
$\qquad$

## Volume - Prisms

A Solid - A three dimensional closed spatial figure.
A Polyhedron - a geometric solid with polygons as faces.

A Face of a Polyhedron - One of the polygons that form the polyhedron. Sometimes these get called sides but the better term is face.

An Edge - The intersection of two faces of a polyhedron.

A Vertex - The intersection of two or more edges.


THE PRISM: is a polyhedron that consists of a polygonal region and its translated image in a parallel plane, with quadrilateral faces connecting the corresponding edges.


RIGHT PRISM: A prism whose lateral faces are perpendicular to the bases.


## Notes Section 15.1



OBLIQUE PRISM: A prism whose lateral faces are NOT ALL perpendicular to the bases.


The two congruent faces that have been translated into parallel planes are called the bases of the prism. The faces that are not based are called the lateral faces. All of these examples are right prisms which mean the base and lateral edges are perpendicular to each other. When working with right prisms the height of the prism is also a lateral edge.

## Volume of a Prism

$V_{\text {prism }}=B h$
where $B$ is the area of the base and $h$ is the height of the prism.

Determine the volume of the following prisms.
1.

2.

3.

4.

5.

6.

$\qquad$

## Volume - Cylinders \& Cones <br> Volume of a Cylinder <br> $V_{\text {Cylinder }}=B h=\pi r^{2} h$



## Cavalieri's Principle

## Bonaventura Fransesco Cavalieri (1598-1647)

If the areas of the cross sections of two solids by any plane parallel to a given plane are invariably equal, then the two solids have the same volume.


A Stack of CD Cases
Cross Section: Square


Stack of Crackers
Cross Section: Circle


Notes Section 15.2
Volume of a Cone
$V_{\text {cone }}=\frac{1}{3} B h=\frac{1}{3} \pi r^{2} h$


Determine the volume of the following.
7.

8.

9.

10.

11.



Volume - Pyramids \& Spheres
Volume of a Pyramid
$V_{\text {Pyramid }}=\frac{1}{3} B h$


Determine the volume of the following.
14.

15.


## Volume of a Sphere



Determine the volume of the following.
16.

17.


22. Find Volume in the Can (not including the 2 tennis balls).

21.


## Chapter 15 Summary

1. Summarize the main idea of the chapter
2. Terms (Include name and definition). Also include key example or picture for each term
3. Theorems and Postulates. Also include key example for each theorem or postulate
4. Key examples of the most unique or most difficult problems from notes, homework or application.
$\qquad$

## Trigonometric Functions - Degree Angles

Notes Section 16.1
Initial Side of an Angle: A ray in an angle that remains fixed.

Terminal Side of an Angle: A ray in an angle that rotates.


Positive Angle: An angle formed by the terminal side rotating counterclockwise.

Negative Angle: An angle formed by the terminal side rotating clockwise.


Standard Position: An angle with its vertex at the origin and its initial side along the positive x -axis.

Quadrantal Angle: An angle in standard position whose terminal side coincides with one of the axes.


Coterminal Angles: Two angles in standard position whose terminal sides coincide with each other.


Degrees: An angle has a measure of one degree if it results from $\frac{1}{360}$ of a complete revolution in the positive direction.

Draw a circle with radius 1 whose center is at the origin. Label each angle around the circle counting by $30^{\circ}$. Do the same for $45^{\circ}$

Ex A: Draw an angle in standard position with the given measure and identify the quadrant in which the terminal sides lies.


Ex $B$ : Find the measure of each angle in degrees.
\#1)
\#2)



Ex C: Find one positive angle and one negative angle that is coterminal with each angle.
\#1) $100^{\circ}$
positive
negative

Ex D: Find a coterminal angle between $0^{\circ}$ and $360^{\circ}$. \#1) $\quad-70^{\circ}$

## Trigonometry - Radian Angles

Radians: The radian measure of an angle in standard position is defined as the length of the corresponding arc on the unit circle.

## Revolutions/Degrees/Radians Relationship

1 Rev $=360^{\circ}=2 \pi$ radians


Draw a circle with radius 1 whose center is at the origin. Label each angle around the circle counting by $\frac{\pi}{6}$ and $\frac{\pi}{4}$.

Ex A: Draw an angle in standard position with the given measure and identify the quadrant in which the terminal sides lies.
\#1) $\frac{5 \pi}{6}$
\#2) $\frac{-11 \pi}{6}$



Ex B: Find the measure of each angle in radians.

> \#1)
\#2)

\#3)
\#4)


Ex C: Find one positive angle and one negative angle that is coterminal with each angle.
\#3) $\frac{7 \pi}{6}$
positive
negative

Ex D: Find a coterminal angle between 0 and $2 \pi$.
\#4) $\quad-\frac{5 \pi}{4}$

Ex E: Convert to radians in terms of $\pi$
\#1) $\quad 135^{\circ}$
\#2)
2) $-45^{\circ}$

Ex F: Convert to degrees
\#1) $3 \pi$
\#2) $-\frac{2 \pi}{3}$

## Trigonometry - Reference Triangle

 Unit Circle - A circle with radius 1 whose center is at the origin.

Reference Triangle - A right triangle in the coordinate plane with an acute angle at the origin and one leg on the $x$-axis.

Notes Section 16.3

1. Find $\sin \theta$

2. If $T(-3,-6)$, find $\cos \theta$

3. Given $\theta$ is in Quadrant IV and $\sin \theta=-\frac{3}{5}$, then find $\cos \theta$.
4. 

5

$$
\sin \theta=
$$

$$
\cos \theta=
$$

$$
\tan \theta=
$$

## Positive Negative Quadrants



What quadrant(s) can $\theta$ lie if...
a. $\sin \theta>0$ and $\cos \theta<0$
b. $\sin \theta$ and $\tan \theta$ have the same sign
$\sin \theta=$
$\cos \theta=$
$\tan \theta=$

Trigonometry - Reference \& Special Angles Notes Section 16.4
Reference Angle - An acute angle formed by the $x$-axis and the terminal side of an angle in standard position.


Find the Reference Angle

$-100^{\circ}$

$$
\frac{23 \pi}{12}
$$

## SPECIAL ANGLES





FIND THE EXACT VALUE!

$\sin 210^{\circ}=$

$\sin \frac{5 \pi}{4}=$
$\cos \left(-\frac{2 \pi}{3}\right)=$


| degrees | radians | $\sin \theta$ | $\cos \theta$ | $\tan \theta$ |
| :---: | :---: | :---: | :---: | :---: |
|  | $\frac{\pi}{2}$ |  |  |  |

## Chapter 16 Summary

1. Summarize the main idea of the chapter
2. Terms (Include name and definition). Also include key example or picture for each term
3. Theorems and Postulates. Also include key example for each theorem or postulate
4. Key examples of the most unique or most difficult problems from notes, homework or application.
