Chapter 8 – Right Triangles

Terms, Postulates and Theorems

Section 8.2

Pythagorean Theorem: In a right triangle, the sum of the squares of the measures of the legs is equals the square of the measure of the hypotenuse.

The converse to the Pythagorean Theorem: If the sum of the squares of the measures of two sides of a triangle equals the square of the measure of the longest side, then the triangle is a right triangle.

Pythagorean Triple: Three whole numbers that satisfy the Pythagorean Theorem. The smallest Pythagorean Triple is the 3 – 4 – 5 triangle.

Section 8.3

Geometric Mean: The geometric mean between two positive numbers, *a* and *b*, is the positive number *x* where $\frac{x}{a} = \frac{b}{x}$.

How will the quiz be structured?

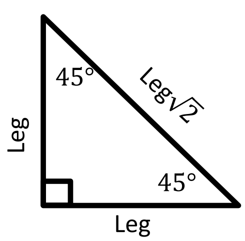
Matching

Theorem 8-1: If the altitude is drawn from the vertex of the right angle of a right triangle to its hypotenuse, then the two triangles formed are similar to the given triangle and each other.

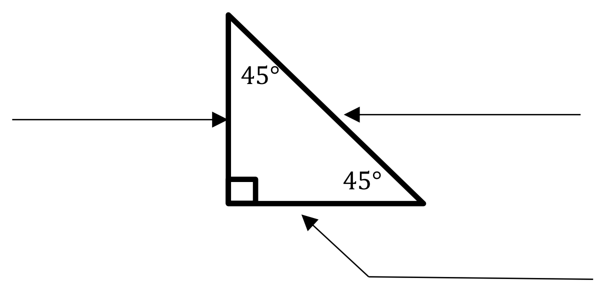
A few key words will be missing from this theorem, and you have to write in the missing words.

Section 8.4

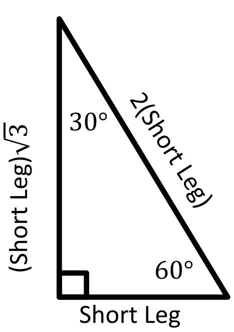
THE 45° - 45° - 90° TRIANGLE (RIGHT ISOSCELES)



Hypotenuse = Leg$\sqrt{2}$

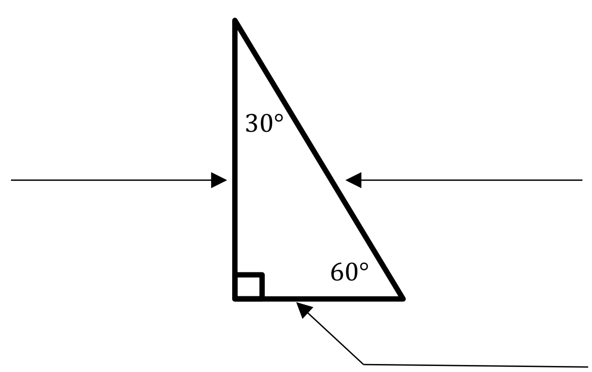


THE 30° - 60° - 90° TRIANGLE



Long Leg = (Short Leg)$\sqrt{3}$
Hypotenuse = 2(Short Leg)

Fill in the words that go on each side of each triangle.



Simplifying Radicals

Notes Section 8.1

Review of Simplifying

- Make a factor bush
- Find perfect squares (or pairs) and square root them to move to outside of radical
- Multiply all inside numbers together and multiply all numbers outside radical together.

Simplify.

1. $\sqrt{18}$

2. $\sqrt{28}$

3. $3\sqrt{27}$

4. $\sqrt{108}$

5. $\sqrt{5^2}$

6. $\sqrt{x^5}$

Review of Multiplying

- First simplify each separate radical if needed
- Then multiply all numbers inside the radical together and all numbers outside the radical together
- Finally simplify again if needed

Multiply. Simplify your answer.

7. $\sqrt{3} \cdot \sqrt{3}$

8. $-\left(\sqrt{3}\right)^2$

9. $\left(-\sqrt{3}\right)^2$

10. $\sqrt{3^2}$

11. $\sqrt{3} \cdot \sqrt{2}$

12. $\sqrt{10} \cdot \sqrt{2}$

Review of Division

- First if possible divide the radicands together and the numbers outside the radical together.
- Then, simplify each separate radical if needed
- Finally, if needed simplify again.

13. $\frac{\sqrt{27}}{\sqrt{3}}$

14. $\frac{\sqrt{48}}{\sqrt{6}}$

15. $\frac{8\sqrt{15}}{5\sqrt{3}}$

16. $\frac{11\sqrt{55}}{\sqrt{11}}$

Rationalize The Denominator

You rationalize when there is a radical in the denominator of the fraction that does not simplify out on its own (like yesterday's division problems).

- First try to simplify with division
- Is there still a radical in the denominator? If so, multiply by 1 in its "clever form of 1". This means to create a fraction that is equivalent to one using that radical.

17. $\frac{1}{\sqrt{3}}$

18. $\frac{1}{\sqrt{2}}$

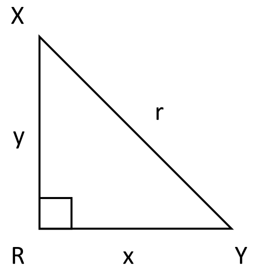
19. $\frac{\sqrt{8}}{\sqrt{3}}$

20. $\frac{\sqrt{11}}{\sqrt{2}}$

Pythagorean Theorem

Notes Section 8.2

Pythagorean Theorem: In a right triangle, the sum of the squares of the measures of the legs is equals the square of the measure of the hypotenuse.



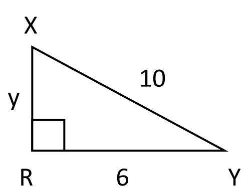
$x^2 + y^2 = r^2$

x and y are always the legs and r is always the hypotenuse.

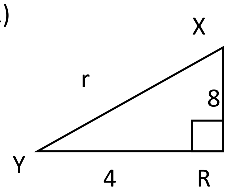
Pythagorean Triple: Three whole numbers that satisfy the Pythagorean Theorem. The smallest Pythagorean Triple is the 3 – 4 – 5 triangle.

Use the Pythagorean Theorem to find the missing measure. Give exact answers and rounded answers (if needed) to one decimal place.

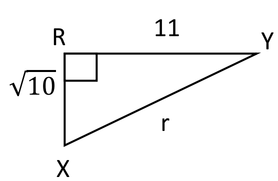
#1)



#2)



#3)



The converse to the Pythagorean Theorem: If the sum of the squares of the measures of two sides of a triangle equals the square of the measure of the longest side, then the triangle is a right triangle.

Determine if the following measures can form a right triangle.

#4) 3, 4, 5

#5) 12, 20, 16

#6) 39, 34, 18

#7) 3.87, 4.47, 5.91

#8) In a right triangle, the measures of the legs are 8 and x + 7, and the measure of the hypotenuse is x + 10. Find the value of x.

#9) The diagonals of a rhombus measure 30 cm and 16 cm. Use the properties of a rhombus and the Pythagorean Theorem to find the perimeter of the rhombus.

Geometric Mean

Notes Section 8.3

Geometric Mean: The geometric mean between two positive numbers, *a* and *b*, is the positive number *x* where $\frac{x}{a} = \frac{b}{x}$.

By multiplying both sides by the denominators, we can see that $x^2 = ab$.

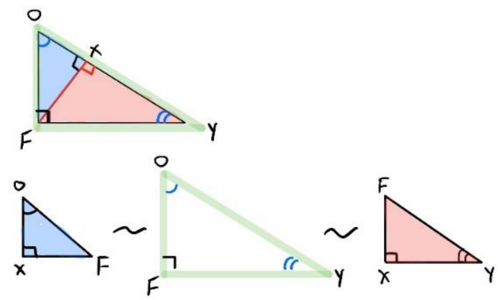
Example of why $x^2 = ab$: Find geometric mean of 5 and 20

Find the geometric mean, x, for each of the following pairs of numbers.

#1) 6 and 27

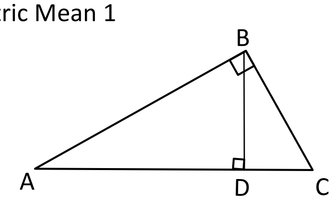
#2) $\frac{3}{2}$ and $\frac{2}{3}$

Theorem 8-1: If the altitude is drawn from the vertex of the right angle of a right triangle to its hypotenuse, then the two triangles formed are similar to the given triangle and each other.

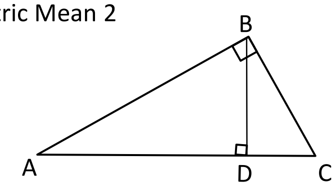


This theorem leads us to 3 specific geometric means.

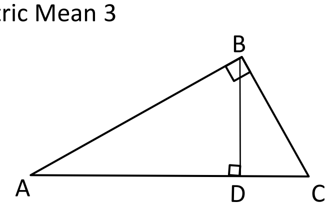
Geometric Mean 1



Geometric Mean 2

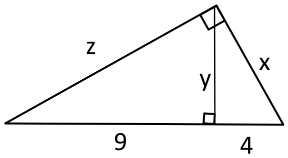


Geometric Mean 3

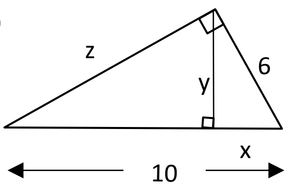


Find the values of x, y and z.

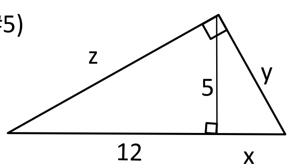
#3)



#4)



#5)

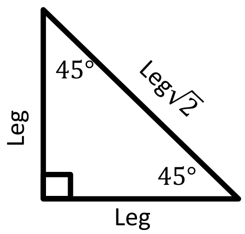


#6) The find the height of the tree in his backyard, KK Slider held the corner of a book near his eye so that the top and bottom of the tree were in line with two edges of the book. If KK's eye is 5 feet off the ground and he is standing 14 feet from the tree, how tall is the tree?

Special Right Triangles

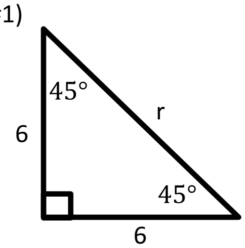
Notes Section 8.4

THE 45° - 45° - 90° TRIANGLE (RIGHT ISOSCELES)

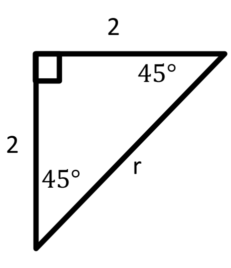


Hypotenuse = Leg$\sqrt{2}$

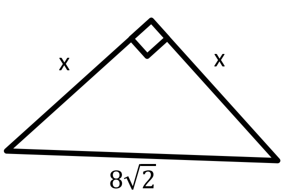
#1)



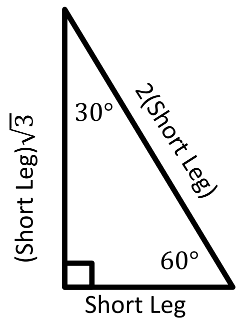
#2)



#3)

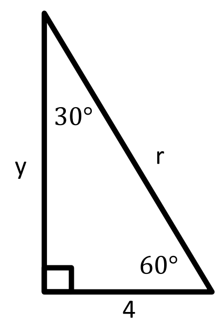


THE 30° - 60° - 90° TRIANGLE

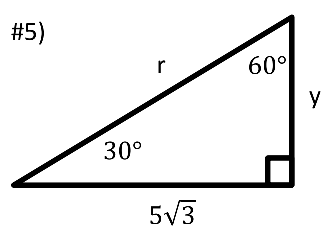


Long Leg = (Short Leg)$\sqrt{3}$
Hypotenuse = 2(Short Leg)

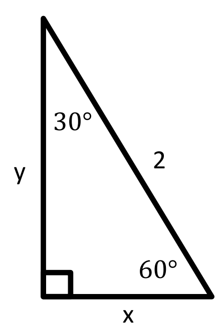
#4)



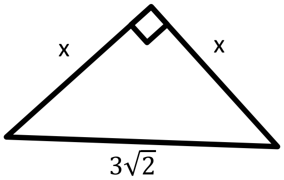
#5)



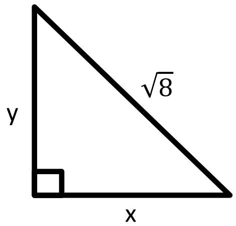
#6)



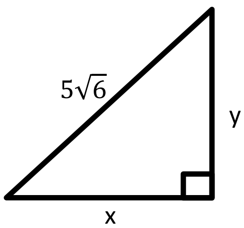
#7)



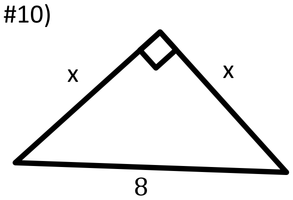
#8)



#9)

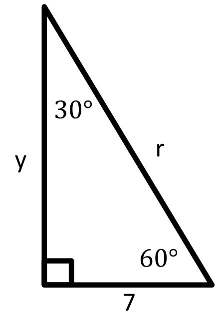


#10)

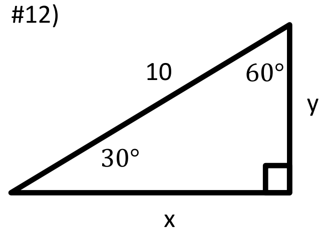


#11) Find the length of a diagonal of a square with sides of 12 inches long.

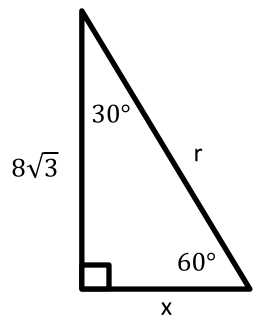
#11)



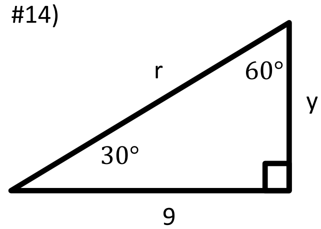
#12)



#13)



#14)



#16) One side of an equilateral triangle measures 20 cm. Find the measure of an altitude of the triangle.

Chapter 8 Summary

1. Summarize the main idea of the chapter

2. Terms (Include name and definition). Also include key example or picture for each term

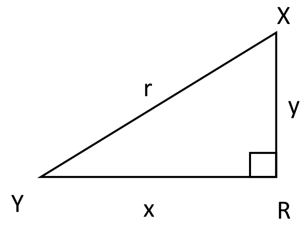
3. Theorems and Postulates. Also include key example for each theorem or postulate

4. Key examples of the most unique or most difficult problems from notes, homework or application.

Chapter 9 – Trigonometry

Terms, Postulates and Theorems

Trigonometric Functions in a Right Triangle: For an acute reference angle Y in right triangle XYR, the trigonometric functions are as follows.

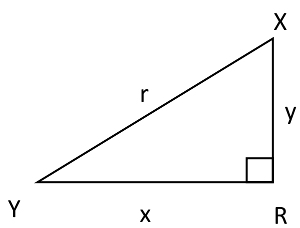


$$\sin(m\angle Y) = \frac{Oppposite\ Leg}{Hypotenuse} = \frac{y}{r}$$

$$\cos(m\angle Y) = \frac{Adjacent\ Leg}{Hypotenuse} = \frac{x}{r}$$

$$\tan(m\angle Y) = \frac{Oppposite\ Leg}{Adjacent\ Leg} = \frac{y}{x}$$

sin = sine
cos = cosine
tan = tangent



$$\sin(m\angle X) = \cos(m\angle Y) \Leftrightarrow \sin(m\angle X) = \cos(90° - m\angle X)$$

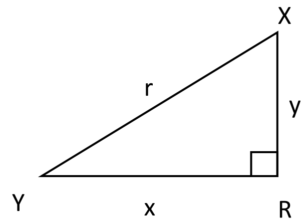
Angle of Elevation = This type of angle starts at a HORIZONTAL line and ELEVATES to form an angle.

Angle of Depression = This type of angle starts at a HORIZONTAL line and DEPRESSES to form an angle.

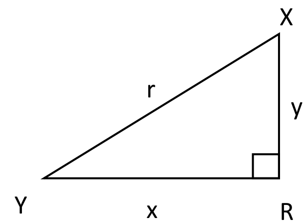
Sine, Cosine, and Tangent
Notes Section 9.1

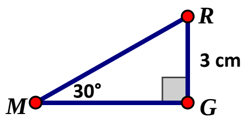
A reference angle must be an acute angle in a right triangle.

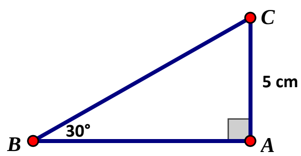
Reference $\angle X$

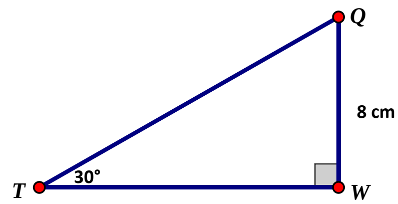


Reference $\angle Y$





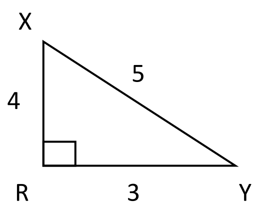




These three ratios have special names.

$\frac{Opposite\ Leg}{Hypotenuse} = \_\_\_\_\_$	$\frac{Opposite\ Leg}{Hypotenuse} = \_\_\_\_\_$	$\frac{Opposite\ Leg}{Hypotenuse} = \_\_\_\_\_$	$= \frac{Opposite\ Leg}{Hypotenuse}$
$\frac{Adjacent\ Leg}{Hypotenuse} = \_\_\_\_\_$	$\frac{Adjacent\ Leg}{Hypotenuse} = \_\_\_\_\_$	$\frac{Adjacent\ Leg}{Hypotenuse} = \_\_\_\_\_$	$= \frac{Adjacent\ Leg}{Hypotenuse}$
$\frac{Opposite\ Leg}{Adjacent\ Leg} = \_\_\_\_\_$	$\frac{Opposite\ Leg}{Adjacent\ Leg} = \_\_\_\_\_$	$\frac{Opposite\ Leg}{Adjacent\ Leg} = \_\_\_\_\_$	$= \frac{Oppposite\ Leg}{Adjacent\ Leg}$

Write a trigonometric function that corresponds to each pair of numbers and the given angle.



#1) 3, 5, $\angle$X

#4) 3, 5, $\angle$Y

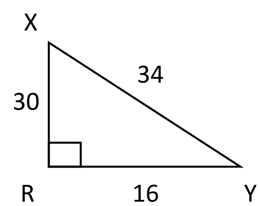
#2) 3, 4, $\angle$X

#5) 3, 4, $\angle$Y

#3) 4, 5, $\angle$X

#6) 4, 5, $\angle$Y

Write an equation using the indicated trig ratio.



#7) $\sin(m\angle X)$

#10) $\sin(m\angle Y)$

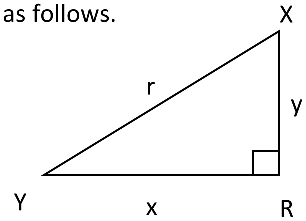
#8) $\cos(m\angle X)$

#11) $\cos(m\angle Y)$

#9) $\tan(m\angle X)$

#12) $\tan(m\angle Y)$

<u>Trigonometric Functions in a Right Triangle</u>: For an acute reference angle Y in right triangle XYR, the trigonometric functions are as follows.



$$\sin(m\angle Y) = \frac{Oppposite\ Leg}{Hypotenuse} = \frac{y}{r}$$

$$\cos(m\angle Y) = \frac{Adjacent\ Leg}{Hypotenuse} = \frac{x}{r}$$

$$\tan(m\angle Y) = \frac{Oppposite\ Leg}{Adjacent\ Leg} = \frac{y}{x}$$

sin = sine
cos = cosine
tan = tangent

SOH-CAH-TOA

r sin A = y

Greek Letters

α =

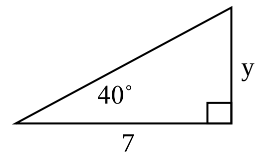
β =

γ =

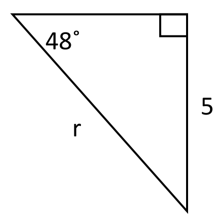
θ =

Find the missing value. Round measures of segments to the nearest tenth and angle measures to the nearest degree.

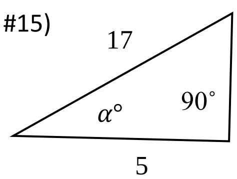
#13)



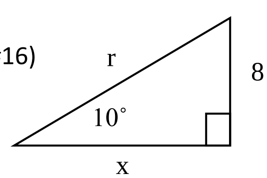
#14)



#15)



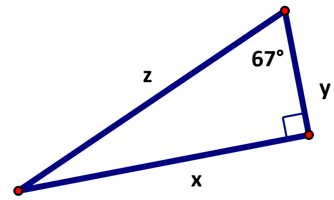
#16)

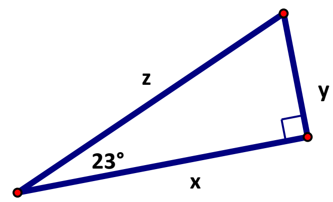


Sine, Cosine and Complementary Angles

Notes Section 9.2

Angle	Sine (sin)	Cosine (cos)	Tangent (tan)		Angle	Sine (sin)	Cosine (cos)	Tangent (tan)
1°	0.0175	0.9998	0.0175		89°	0.9998	0.0175	57.290
2°	0.0349	0.9994	0.0349		88°	0.9994	0.0349	28.636
3°	0.0523	0.9986	0.0524		87°	0.9986	0.0523	19.081
4°	0.0698	0.9976	0.0699		86°	0.9976	0.0698	14.300
5°	**0.0872**	0.9962	0.0875		85°	0.9962	**0.0872**	11.430
6°	0.1045	0.9945	0.1051		84°	0.9945	0.1045	9.5144
7°	0.1219	0.9925	0.1228		83°	0.9925	0.1219	8.1443
8°	0.1392	0.9903	0.1405		82°	0.9903	0.1392	7.1154
9°	0.1564	0.9877	0.1584		81°	0.9877	0.1564	6.3138
10°	**0.1736**	0.9848	0.1763		80°	0.9848	**0.1736**	5.6713
11°	0.1908	0.9816	0.1944		79°	0.9816	0.1908	5.1446
12°	0.2079	0.9781	0.2126		78°	0.9781	0.2079	4.7046
13°	0.2250	0.9744	0.2309		77°	0.9744	0.2250	4.3315
14°	0.2419	0.9703	0.2493		76°	0.9703	0.2419	4.0108
15°	0.2588	0.9659	0.2679		75°	0.9659	0.2588	3.7321
16°	0.2756	0.9613	0.2867		74°	0.9613	0.2756	3.4874
17°	0.2924	0.9563	0.3057		73°	0.9563	0.2924	3.2709
18°	0.3090	0.9511	0.3249		72°	0.9511	0.3090	3.0777
19°	0.3256	0.9455	0.3443		71°	0.9455	0.3256	2.9042
20°	0.3420	0.9397	0.3640		70°	0.9397	0.3420	2.7475
21°	0.3584	0.9336	0.3839		69°	0.9336	0.3584	2.6051
22°	0.3746	0.9272	0.4040		68°	0.9272	0.3746	2.4751
23°	**0.3907**	0.9205	0.4245		67°	0.9205	**0.3907**	2.3559
24°	0.4067	0.9135	0.4452		66°	0.9135	0.4067	2.2460
25°	0.4226	0.9063	0.4663		65°	0.9063	0.4226	2.1445
26°	0.4384	0.8988	0.4877		64°	0.8988	0.4384	2.0503
27°	0.4540	0.8910	0.5095		63°	0.8910	0.4540	1.9626
28°	0.4695	0.8829	0.5317		62°	0.8829	0.4695	1.8807
29°	0.4848	0.8746	0.5543		61°	0.8746	0.4848	1.8040
30°	0.5000	0.8660	0.5774		60°	0.8660	0.5000	1.7321
31°	0.5150	0.8572	0.6009		59°	0.8572	0.5150	1.6643
32°	0.5299	0.8480	0.6249		58°	0.8480	0.5299	1.6003
33°	**0.5446**	0.8387	0.6494		57°	0.8387	**0.5446**	1.5399
34°	0.5592	0.8290	0.6745		56°	0.8290	0.5592	1.4826
35°	0.5736	0.8192	0.7002		55°	0.8192	0.5736	1.4281
36°	0.5878	0.8090	0.7265		54°	0.8090	0.5878	1.3764
37°	0.6018	0.7986	0.7536		53°	0.7986	0.6018	1.3270
38°	0.6157	0.7880	0.7813		52°	0.7880	0.6157	1.2799
39°	0.6293	0.7771	0.8098		51°	0.7771	0.6293	1.2349
40°	0.6428	0.7660	0.8391		50°	0.7660	0.6428	1.1918
41°	0.6561	0.7547	0.8693		49°	0.7547	0.6561	1.1504
42°	0.6691	0.7431	0.9004		48°	0.7431	0.6691	1.1106
43°	0.6820	0.7314	0.9325		47°	0.7314	0.6820	1.0724
44°	0.6947	0.7193	0.9657		46°	0.7193	0.6947	1.0355
45°	**0.7071**	**0.7071**	1.0000					





Find the value of x.

#1) $\sin(x) = \cos(23°)$

#2) $\sin(65°) = \cos(x)$

#3) $\sin(30°) = \cos(x)$

#4) $\sin(x) = \cos(60°)$

#5) $\sin(x) = \cos(45°)$

#6) $\sin(2x + 1)° = \cos(40°)$

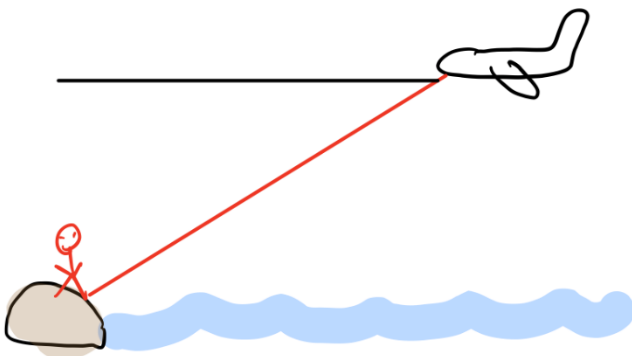
#7) $\sin(x - 10)° = \cos(6x + 40)°$

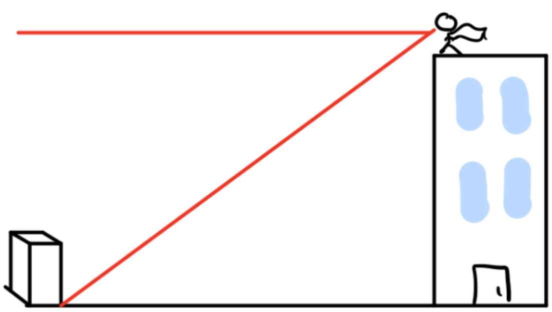
#8) $\sin\left(\frac{1}{2}x - 5\right)° = \cos(x - 30)°$

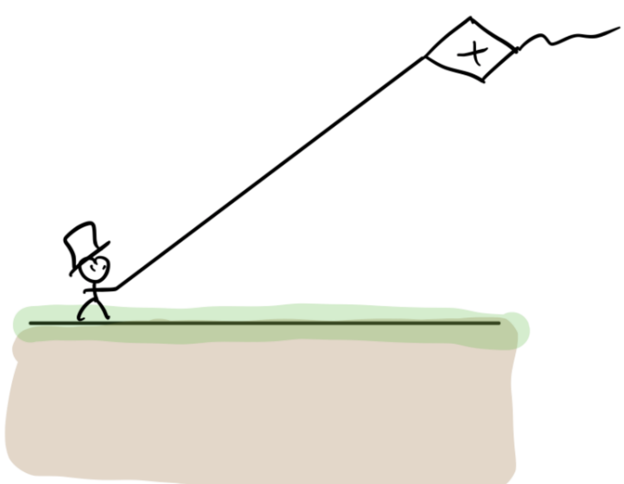
Trigonometry Applications

Notes Section 9.3

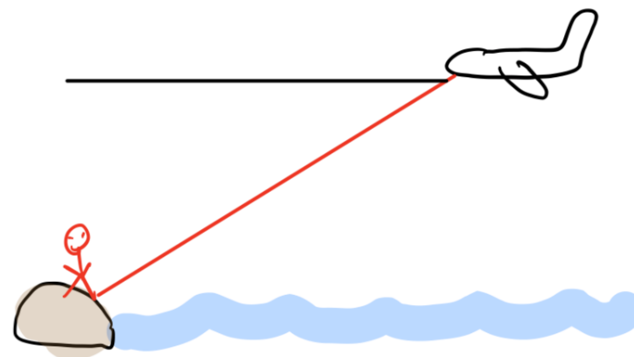
Angle of Elevation = This type of angle starts at a HORIZONTAL line and ELEVATES to form an angle.

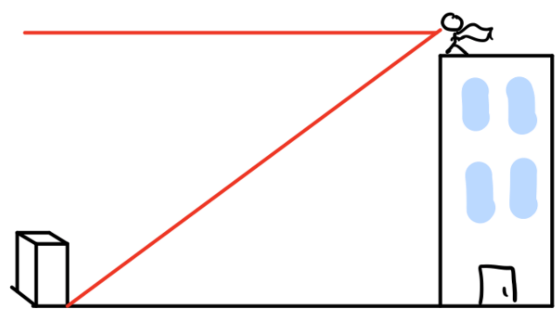


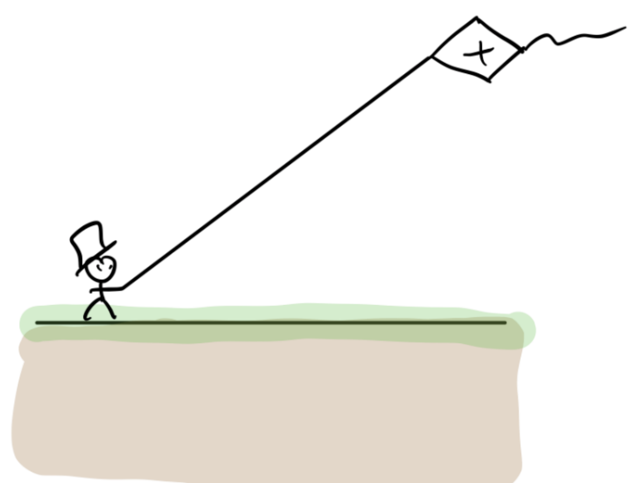




Angle of Depression = This type of angle starts at a HORIZONTAL line and DEPRESSES to form an angle.







Solve each problem. If needed, round measures of segments to the nearest hundredth and measures of angles to the nearest degree. You must draw a picture.

#1) At a certain time of day, the angle of elevation of the sun is 24°. Find the length of a shadow cast by a building 90 feet high.

#2) Narcoleptic Nelly is flying a kite while taking a nap. The string is 50 meters long and forms an angle of 45° with the ground. How high is the kite above the ground?

#3) George decides to take all his headless dolls and chuck them into a river. He wants to make sure that when the dolls hit the water's surface they become totally submerged in the river water, so he intends on climbing to the very top of a bridge. So George sets out to find an appropriate bridge to hurl his headless dolls off. Upon walking somewhat aimlessly in search of a bridge that is *just* right, George finds himself standing 100 meters from a bridge made of ginger bread and honey. "Mmmm, ginger bread and honey," George mumbles to himself. From his standing position, he determines that the angle of elevation to the top of the delicious bridge is 35°. George's eye level is 1.45 meters above the ground. Find the height of the bridge.

#4) From the top of a lighthouse Hazel Nut can see something floating in the open sea. Using her binoculars, she can clearly see that the floating object is in fact a floating, headless doll. The angle of depression to the floating, headless doll is 25°. If the top of the light house is 150 feet above sea level, find the distance from the doll to the foot of the lighthouse.

Trigonometry & Systems of Equations Notes Section 9.4

#1) A homeless giant is at the top of a building. 200 feet from the base of the building, the angle of elevation of the top of the hobo is 32° and the angle of elevation of the bottom of the hobo is 30°. Determine the height of the hobo (to the nearest foot).

#2) In a rubber ducky floaty 400 feet from the base of the Cliffs of Insanity, George sees the base of the Starbucks at 18° and the top of the Starbucks at 21°. How tall is the Starbucks (to the nearest foot)?

#3) George and his paradoxasaur are on either side of a giant steamy pile of paradoxasaur poop and are 40 feet apart. George sees the top of the poop at 42° and his paradoxasaur sees the top of the poop at 36°. How high is the pile of poop (to the nearest foot)?

#4) On a sightseeing trip to the garbage dump, George spots a mound of Atari ET cartridges at 22° and Cathy spots the same mound at 30°. If the two nitwits are 310 feet apart, determine the height of the mound (to the nearest foot).

Chapter 9 Summary

1. Summarize the main idea of the chapter

2. Terms (Include name and definition). Also include key example or picture for each term

3. Theorems and Postulates. Also include key example for each theorem or postulate

4. Key examples of the most unique or most difficult problems from notes, homework or application.

Chapter 10 – Law of Sines & Cosines

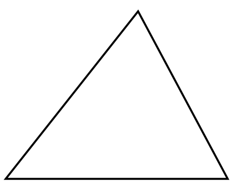
Terms, Postulates and Theorems

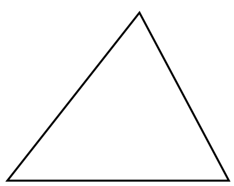
Law of Sines: Let ΔABC be any triangle with a, b, and c representing the measures of sides opposite angles with measures A, B, and C respectively. Then,

$$\frac{\sin(m\angle A)}{a} = \frac{\sin(m\angle B)}{b} = \frac{\sin(m\angle C)}{c}$$

The Law of Sines can be used to solve a triangle in the following cases:

1. You are given the measure of two angles and any side of a triangle.
2. You are given the measure of two sides and an angle opposite one of these sides of the triangle.





Solving the Triangle: Finding the measures of all the angles and sides of a triangle.

Law of Cosines: Let ΔABC be any triangle with a, b, and c representing the measures of sides opposite angles with measures A, B, and C respectively. Then, the following equations hold true.

$a^2 = b^2 + c^2 – 2bc \cos (m\angle A)$

$b^2 = a^2 + c^2 – 2ac \cos (m\angle B)$

$c^2 = a^2 + b^2 – 2ab \cos (m\angle C)$

The law of cosines can be used to solve a triangle in the following cases.

1. To find the measure of the third side of any triangle if the measures of the two sides and the included angle are given.
2. To find the measure of an angle of a triangle if the measures of the three sides are given. (If you are given SSS, **YOU CANNOT USE SINES TO FIND THE *LARGEST* ANGLE.**)

Solving Complex Equations

Notes Section 10.1

Solve each equation showing all your work. Round angles to the nearest tenth and segments to the nearest hundredth

#1) $b^2 = a^2 + c^2 - 2ac \cos (m\angle B)$
$15^2 = 10^2 + 6^2 - 2(10)(6) \cos (m\angle B)$

#2) $a^2 = b^2 + c^2 - 2bc \cos \cos (m\angle A)$
$a^2 = 6^2 + 4^2 - 2(6)(4) \cos (20^o)$

#3) $c^2 = a^2 + b^2 - 2ab \cos \cos (m\angle C)$
$5^2 = 3^2 + 4^2 - 2(3)(4) \cos (m\angle C)$

#4) $b^2 = a^2 + c^2 - 2ac \cos (m\angle B)$
$b^2 = 3^2 + 8^2 - 2(3)(8) \cos (40^o)$

Law of Sines

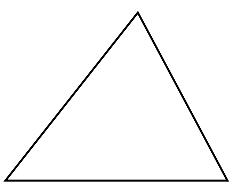
Notes Section 10.2

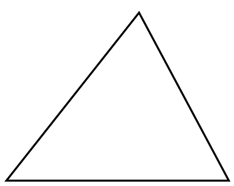
Law of Sines: Let ΔABC be any triangle with a, b, and c representing the measures of sides opposite angles with measures A, B, and C respectively. Then,

$$\frac{\sin(m\angle A)}{a} = \frac{\sin(m\angle B)}{b} = \frac{\sin(m\angle C)}{c}$$

The Law of Sines can be used to solve a triangle in the following cases:

3. You are given the measure of two angles and any side of a triangle.
4. You are given the measure of two sides and an angle opposite one of these sides of the triangle.





Solving the Triangle: Finding the measures of all the angles and sides of a triangle.

For the following examples, round the sides to the nearest tenth and the angles to the nearest whole number.

#1) Solve ΔABC if m∠A = 50°, m∠B = 67°, and b = 10.

#2) If $a = 10$, $m\angle C = 124°$, and $c = 25$, find $m\angle A$.

#3) Two of George's paradoxasaurs, Bert and Ernie, fly away from George at the same time. Both paradoxasaurs travel at a speed of 50 miles per hour. Bert flies in the direction of 50° west of north while Ernie travels 10° west of south. How far apart are Bert and Ernie after 4 hours?

Law of Cosines

Notes Section 10.3

Law of Cosines: Let ΔABC be any triangle with a, b, and c representing the measures of sides opposite angles with measures A, B, and C respectively. Then, the following equations hold true.

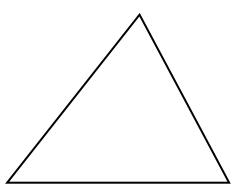
$$a^2 = b^2 + c^2 - 2bc\cos(m\angle A)$$

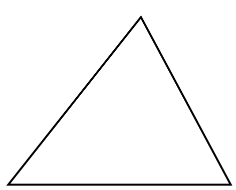
$$b^2 = a^2 + c^2 - 2ac\cos(m\angle B)$$

$$c^2 = a^2 + b^2 - 2ab\cos(m\angle C)$$

The law of cosines can be used to solve a triangle in the following cases.

3. To find the measure of the third side of any triangle if the measures of the two sides and the included angle are given.
4. To find the measure of an angle of a triangle if the measures of the three sides are given. (If you are given SSS, **YOU CANNOT USE SINES TO FIND THE *LARGEST* ANGLE.**)





For the following examples, round the sides to the nearest tenth and the angles to the nearest whole number.

#1) In ΔABC if a = 20, c = 24, and m∠B = 47°, find b.

#2) In ΔABC if $a = 5$, $b = 6$, and $c = 7$, find $m\angle C$.

#3) George is 20 inches from Rickito and 100 inches from Danny Devito. The angle formed by the two and George is 30°. How many inches apart are Rickito and Danny Devito?

Chapter 10 Summary

1. Summarize the main idea of the chapter

2. Terms (Include name and definition). Also include key example or picture for each term

3. Theorems and Postulates. Also include key example for each theorem or postulate

4. Key examples of the most unique or most difficult problems from notes, homework or application.

Chapter 11 – Completing Circle Squares

Terms, Postulates and Theorems

Factor by grouping

Difference of Two Squares

$$x^2 - a^2 = (x + a)(x - a)$$

Difference/Sum of Two Cubes

$$x^3 \pm a^3 = (x \pm a)(x^2 \mp ax + a^2)$$

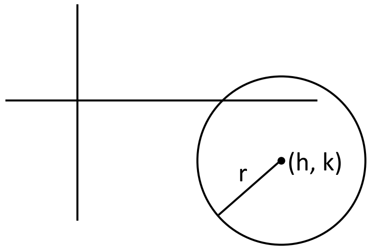
Perfect Square Trinomial

$$x^2 \pm 2ax + a^2 = (x \pm a)^2$$

Splitting the Middle Term

Standard equation of a circle: In general, an equation for a circle with center at *(h, k)* and a radius of *r* units is

$$(x - h)^2 + (y - k)^2 = r^2$$



Completing the Square

Factoring Perfect Square Trinomial Review

Notes Section 11.1

Pre-steps:

1) Write terms in descending order with respect to one of the variables.
2) Make sure lead coefficient is positive.

Factoring Review

Notes Section 11.2

Pre-steps:

1) Write terms in descending order with respect to one of the variables.
2) Make sure lead coefficient is positive.

Four Terms

1) Factor out GCF

$aw \pm ax \pm ay \pm az = a[w \pm x \pm y \pm z]$

2) Factor by grouping

Binomial

1) Factor out GCF

$ax \pm ay = a[x \pm y]$

2) Difference of Two Squares

$x^2 - a^2 = (x + a)(x - a)$

3) Difference/Sum of Two Cubes

$x^3 \pm a^3 = (x \pm a)(x^2 \mp ax + a^2)$

Trinomials

1) Factor out GCF

$ax \pm ay \pm az = a[x \pm y \pm z]$

2) Perfect Square Trinomial 11.1

$x^2 \pm 2ax + a^2 = (x \pm a)^2$

3) Splitting the Middle Term

Factor each polynomial.

#1) $-2 + x^3 - x^2 + 2x$

#2) $-192x^2y - 72x^3 + 24rxy + 9rx^2$

Factor each binomial.

#3) $200 - 98x^2$

#4) $49x^2 - 100$

#5) $49x(x + 4) - 100(x + 4)$

#6) $x^2(x - 10) + 17(x - 10)$

Factor each using perfect square trinomial.

#7) $10x^2 + 100x + 250$

#8) $49x^2 - 56x + 16$

Factor by the Australian method

#9) $19x + 5x^2 + 12$

#10) $-16x^2 - 60x + 100$

Factor each using the difference or sum of two cubes.

#11) $1029x^3y - 24y^4$

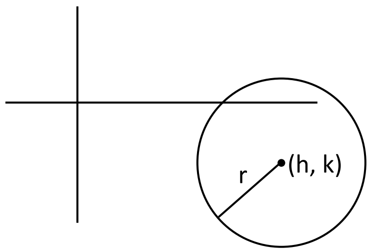
#12) $-1 - x^3$

Equations of a Circle

Notes Section 11.3

Standard equation of a circle: In general, an equation for a circle with center at *(h, k)* and a radius of *r* units is

$$(x-h)^2 + (y-k)^2 = r^2$$



Determine the coordinates of the center and the measure of the radius for each circle whose equation is given.

#1) $(x - 7)^2 + (y - 4)^2 = 6^2$

Center =

Radius =

#2) $(x + 5)^2 + (y + 11)^2 = 8^2$

Center =

Radius =

#3) $(x - 12)^2 + (y + 17)^2 = 100$

Center =

Radius =

#4) $(x + 21)^2 + (y - 41)^2 = 49$

Center =

Radius =

#5) $(x-2)^2 + (y-1)^2 = \sqrt{81}$

Center =

Radius =

#6) $(x+1)^2 + \left(y+\sqrt{2}\right)^2 = 98$

Center =

Radius =

The coordinates of the center and the measure of the radius of a circle are given. Write an equation of the circle.

#7) (4, 9), 8

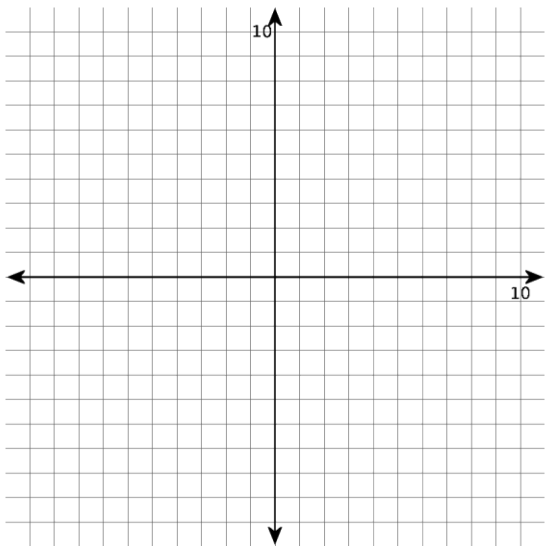
#8) (-5, -8), 11

#9) (-3, 6), $\sqrt{2}$

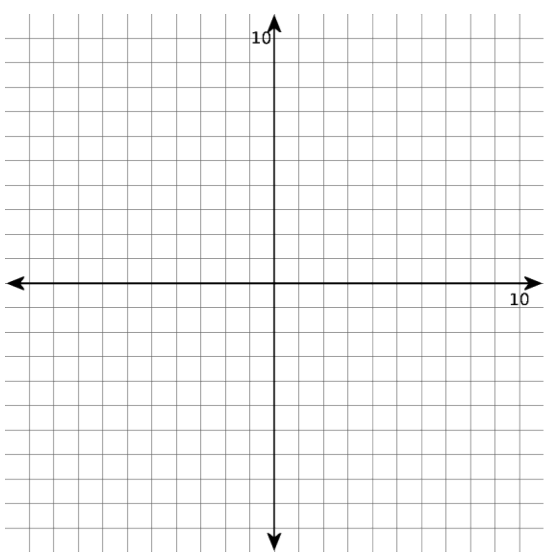
#10) (14, -19), $\sqrt{10}$

Graph each equation.

#11) $(x + 1)^2 + (y - 2)^2 = 9$



#12) $x^2 + (y + 6)^2 - 25 = 0$



Completing the Square

Notes Section 11.4

Review Perfect Square Trinomial

What makes a trinomial a perfect square?

$$(x-3)^2 = x^2 - 6x + 9$$

$$(x+4)^2 = x^2 + 8x + 16$$

$$(2x-5)^2 = 4x^2 - 20x + 25$$

$$x^2 + 20x + 100 = (x+10)^2$$

$$x^2 + 14x + 7 = (x+49)^2$$

$$25x^2 + 60x + 36 = (5x+6)^2$$

Complete these Perfect Square Trinomials

x^2 _______ $+ 25$

x^2 _______ $+ 100$

x^2 _______ $+ 121$

$x^2 - 4x +$ ______

$x^2 + 8x +$ _______

$x^2 + 16x +$ _______

Completing the square when a = 1

$$ax^2 + bx + c = 0$$

$$x^2 + bx + c = 0$$

Find the constant that would complete each square.

#1) $x^2 + 2x = 0$

#2) $x^2 + 10x - 8 = 0$

#3) $x^2 + 14x - 1 = 0$

#4) $x^2 + 15x = 12$

#5) $x^2 + 1x - 14 = 0$

Completing Circle Squares

Notes Section 11.5

Standard equation of a circle: In general, an equation for a circle with center at *(h, k)* and a radius of *r* units is

$$(x-h)^2+(y-k)^2=r^2$$

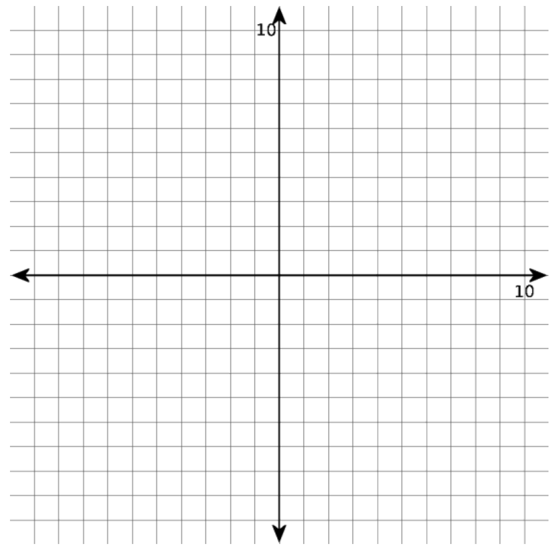
Write each equation of a circle in standard form by completing some squares. Identify the center and radius.

#1) $x^2-6x+y^2-2y-8=0$

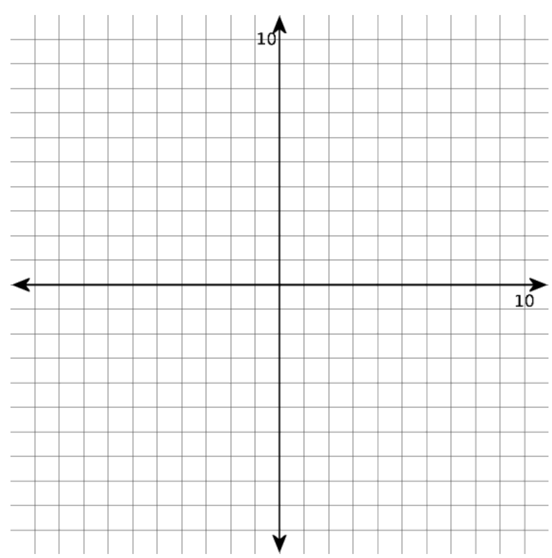
#2) $x^2-8x+y^2+10y=10$

Identify the center and radius, then graph each circle.

#1) $x^2-6x+y^2+4y-3=0$



#2) $x^2+16x+y^2+12y=-91$



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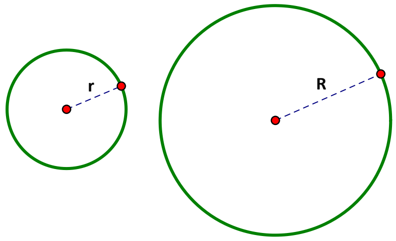
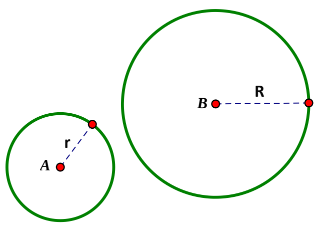
Circle Transformations

Notes Section 12.1

ALL CIRCLES ARE SIMILAR.



Show how Circle A is similar to Circle B by using similarity transformations.



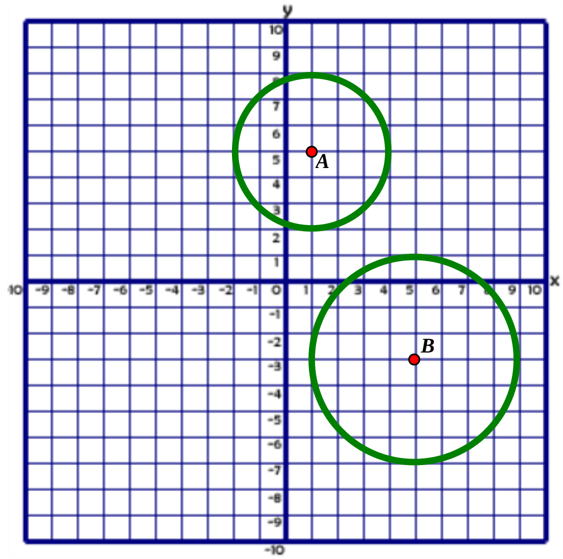
Given Circle A and Circle B with radii, r and R, respectively.

Translate Circle A by vector $\overrightarrow{AB}$. This will create concentric circles.

Dilate circle A by a factor of $\frac{R}{r}$.

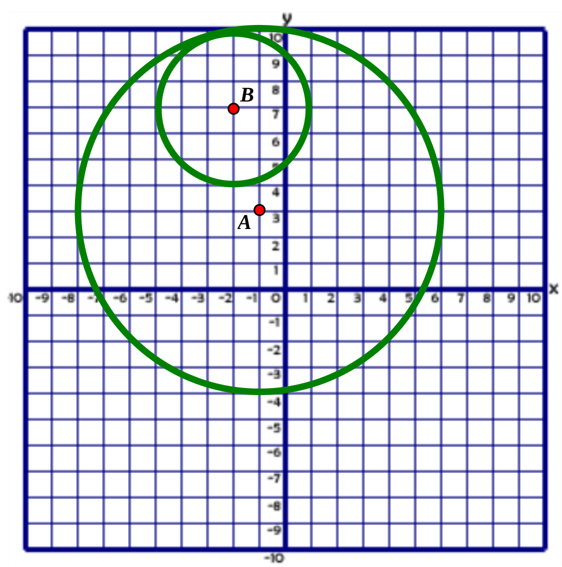
Determine the translation vector and scale factor of the dilation for the following similarity transformations.

Circle A to Circle B



Translate Vector <_____ , _____> , then $D_{B,}$___$(\odot A) = \odot B$

Circle B to Circle A



Translate Vector <_____ , _____> , then $D_{A,}$___$(\odot B) = \odot A$

Circle Terminology

Section 12.2

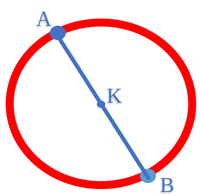
Circle: a set of all points in a plane that are a given distance from a given point in the plane.

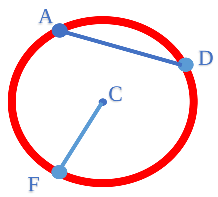
Center: the point in the middle of the circle in which all points in the plane are equidistant.

Chord: a segment that has endpoints on a circle.

Diameter: a chord that contains the center of the circle.

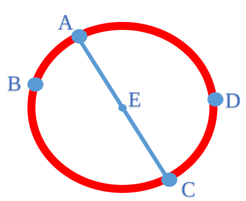
Radius: a segment with one endpoint at the center of a circle and the other endpoint on the circle.





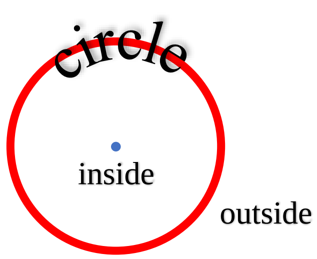
Arc: an unbroken part of a circle.

- *Minor Arc*: an arc that measures less than 180.
- *Major Arc*: an arc that measures more than 180.
- *Semicircle*: an arc that measures 180.



A circle separates a plane into three parts:

the interior, the exterior, and the circle itself.

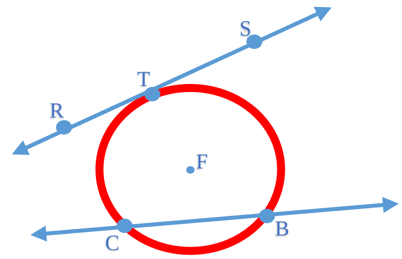


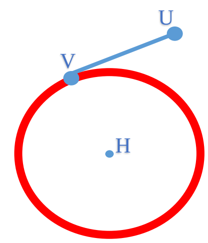
Tangent: a line that intersects a circle in exactly one point.

Point of Tangency: The point at which a tangent line intersects a circle

Tangent Segment: A segment that intersects a circle exactly once and if extended would still only intersect it once.

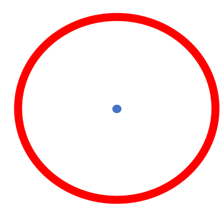
Secant: a line that intersects a circle in exactly two points.





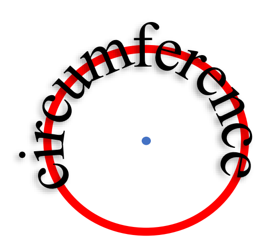
AREA OF A CIRCLE

$A_{\odot} = \pi r^2$

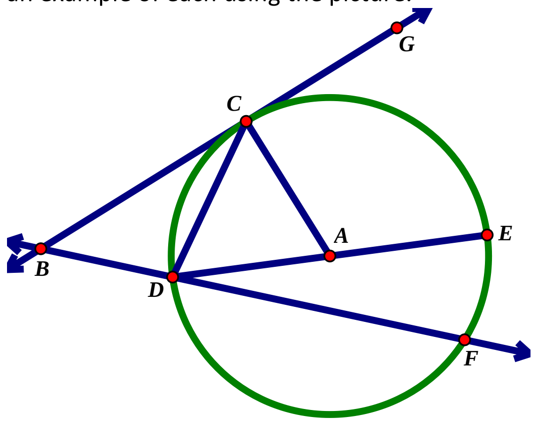


CIRCUMFERENCE (PERIMETER)

$C = 2r\pi = d\pi$



Give an example of each using the picture.



Radius

Diameter

Chord

Secant

Minor Arc

Tangent

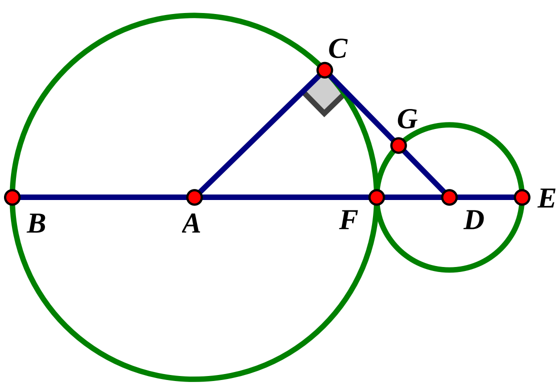
Center

Exterior Point

Major Arc

Semi-Circle

Circles A and D have radii of 4 cm & 1 cm respectively.
Use this information to determine the missing values.



BF = __________

AD = __________

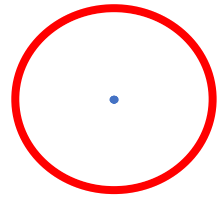
CD = __________

CG = __________

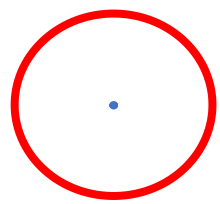
Perimeter of ΔACD = __________

Draw the following relationships.

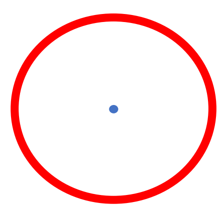
Secant line $\overleftrightarrow{AB}$ intersects $\odot M$ at points A and B.



Secant line $\overleftrightarrow{MN}$ intersects tangent line $\overleftrightarrow{TM}$ on Circle R.



Diameter $\overline{AB}$ intersects tangent line $\overleftrightarrow{GB}$ on circle M.

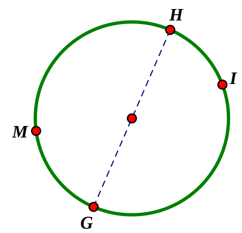


Circles' Central Angles & Arcs

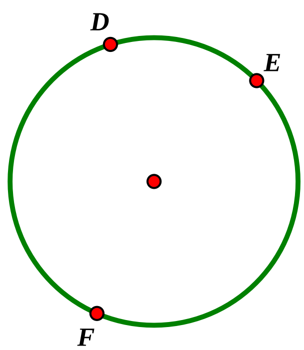
Notes Section 12.3

Arc: an unbroken part of a circle.

- *Minor Arc*: an arc that measures less than 180.
- *Major Arc*: an arc that measures more than 180.
- *Semicircle*: an arc that measures 180.

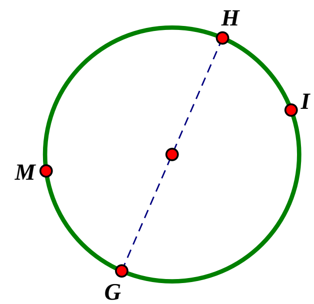


Name each of the following from the picture.

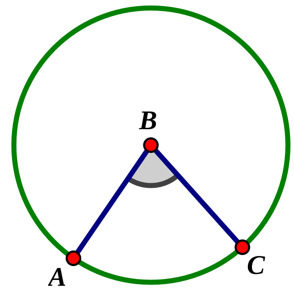


Minor Arc Major Arc Semicircle

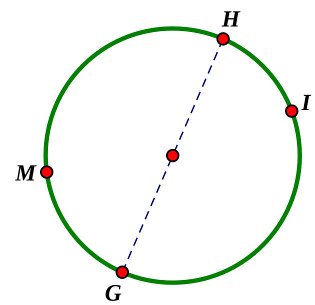
Arc Length (Distance) & Arc Angle (Angle Measure)



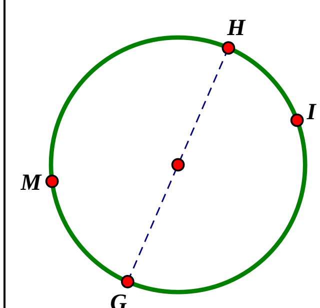
Adjacent Arcs: arcs of a circle that have exactly one point in common.



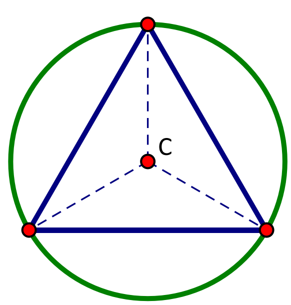
Arc Measure: the measure of a arc is the measure of its central angle. The measure of a semicircle is 180.



Arc Addition Postulate: The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.



Theorem 12.1: In the same (or in congruent) circle, two arcs are congruent IFF their corresponding central angles are congruent.

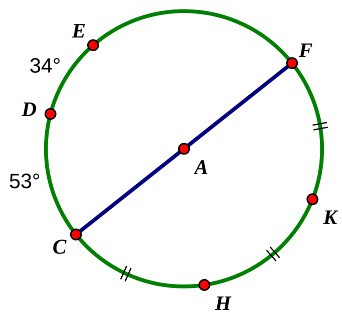


Central angle of a regular polygon.

$$m\angle C = \frac{360}{n}$$

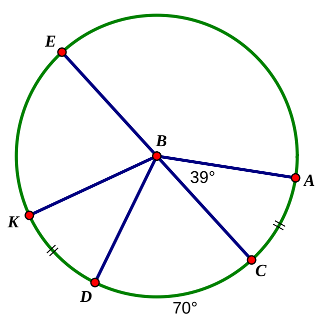
where n is the number of sides and $\angle C$ is the central angle.

Complete each equation.



$m\widehat{CE} =$ $m\widehat{EF} =$

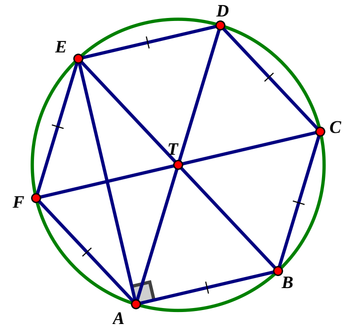
$m\widehat{ECK} =$ $m\widehat{DFC} =$



$m\widehat{AC} =$ $m\widehat{AE} =$

$m\widehat{EK} =$ $m\angle KBD =$

Given a regular polygon, complete each equation.

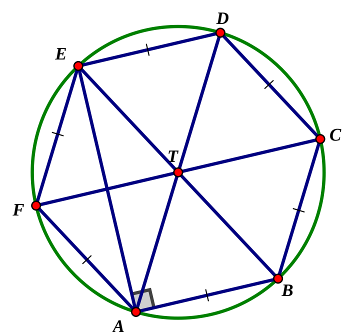


$m\angle ATB =$ $m\angle DTB =$

$m\widehat{AC} =$ $m\widehat{ECA} =$

$m\angle AEB =$ If $AB = 5$ cm, what does $TB =$

If $AB = 5$ cm, what does $EA =$



Chapter 12 Summary

1. Summarize the main idea of the chapter

2. Terms (Include name and definition). Also include key example or picture for each term

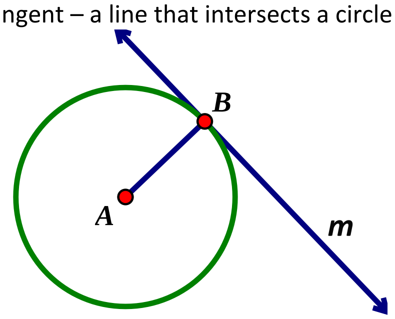
3. Theorems and Postulates. Also include key example for each theorem or postulate

4. Key examples of the most unique or most difficult problems from notes, homework or application.

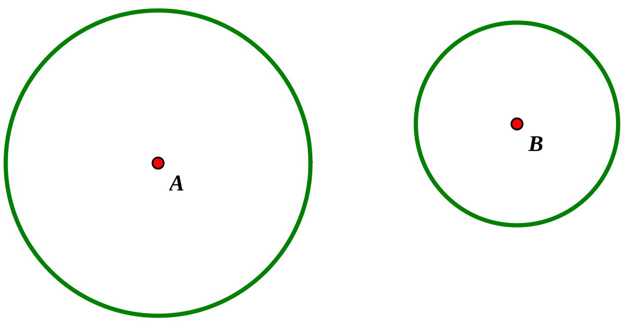
Tangent

Notes 13.1

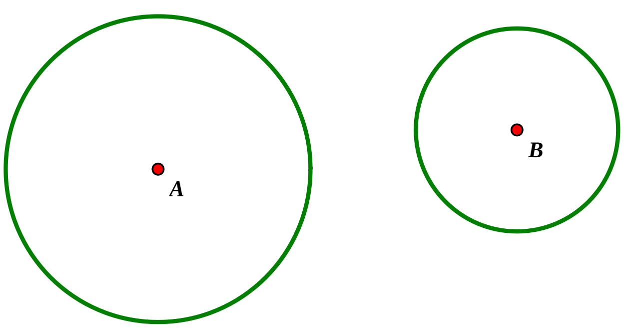
Tangent – a line that intersects a circle once



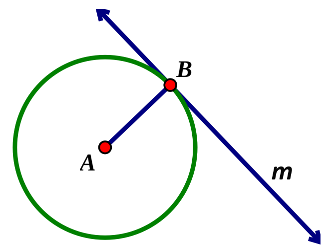
Internally Common Tangent Lines



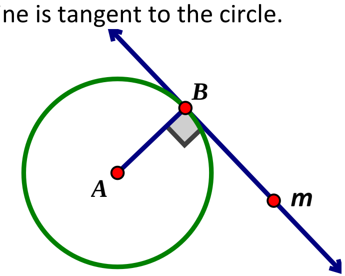
Externally Common Tangent Lines



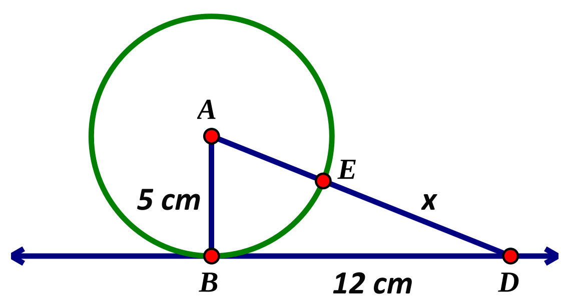
<u>Theorem 13.1</u> – If a line is tangent to a given circle, then the tangent line is perpendicular to the radius at the point of tangency.



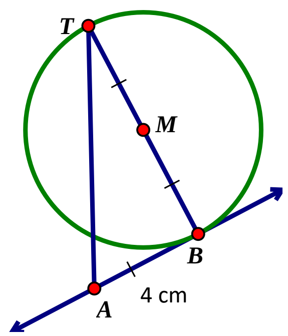
<u>Converse of the Theorem 13.1</u> -- If a line is perpendicular to a radius of a circle at its endpoint on the circle, then the line is tangent to the circle.



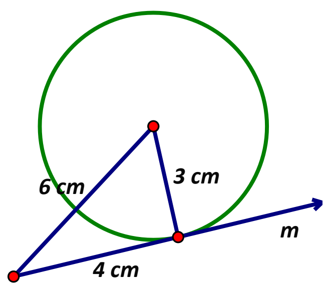
Given that $\overleftrightarrow{BD}$ is a tangent line and that the radius of circle A is 5 cm and BD = 12 cm, determine ED?



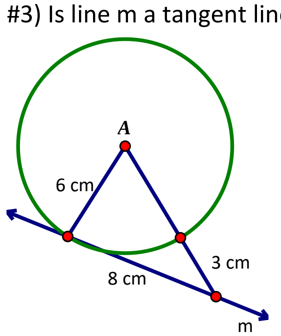
#1) Solve for the requested information, given the $\overleftrightarrow{AB}$ is a tangent line to circle M. Find AT (2 decimals)



#2) Is line m a tangent line?



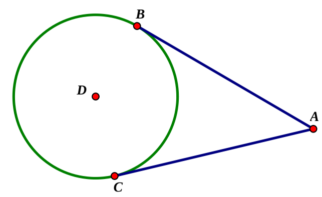
#3) Is line m a tangent line to circle A?

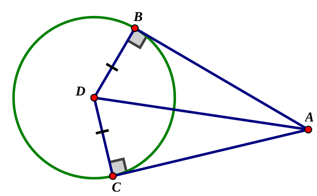


#4) What is the radius of the circle?

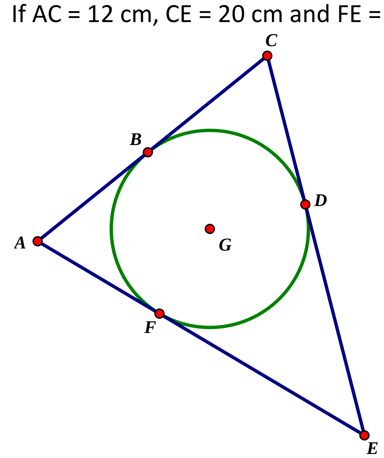
$$x^2 + y^2 - 10x + 8y + 16 = 0$$

Theorem 13.2 – If two segments from the same exterior point are tangent to a circle, then they are congruent to each other.

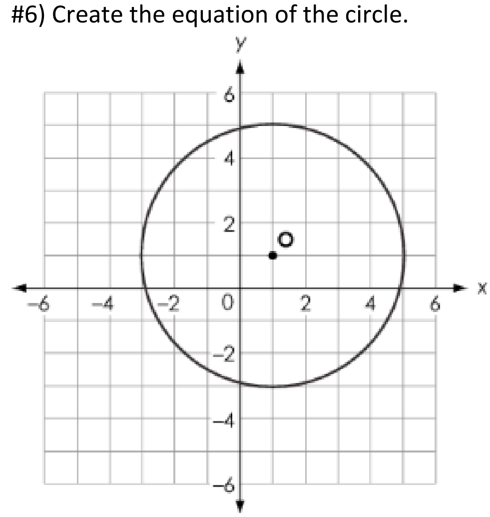




#5) The three segments are tangent at point B, F, and D. If AC = 12 cm, CE = 20 cm and FE = 13 cm, determine AF?



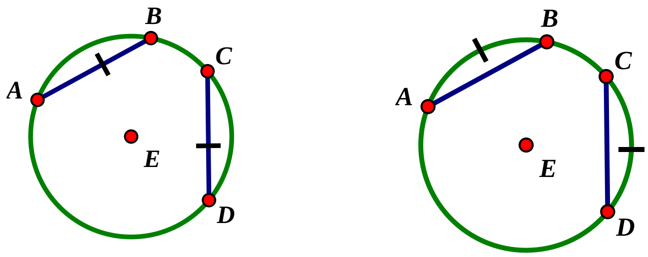
#6) Create the equation of the circle.



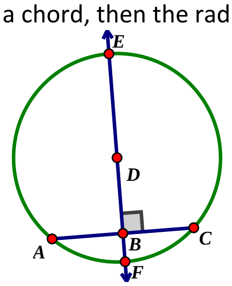
Chord Theorems

Notes Section 13.2

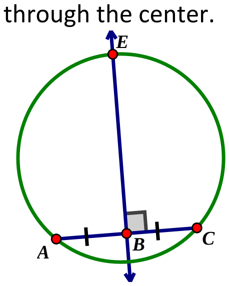
Theorem 13.2: Two cords are congruent, IFF their corresponding arcs are congruent.



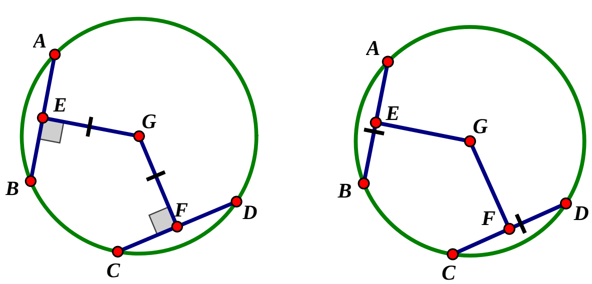
Theorem 13.4: If a radius (or diameter) is perpendicular to a chord, then the radius bisects the chord and arc.



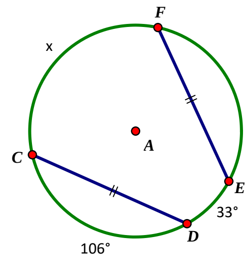
Theorem 13.5: If a segment (or diameter) is the perpendicular bisector of a chord, then the segment goes through the center.



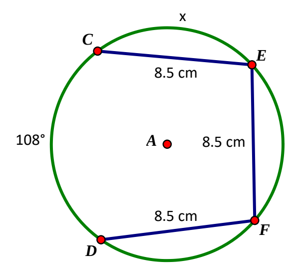
Theorem 13.6: Two chords are equidistant from the center of a circle IFF the chords are congruent.



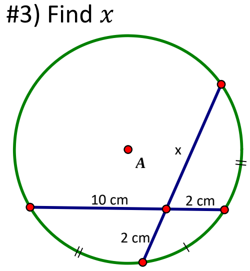
#1) Find $m\widehat{CF}$



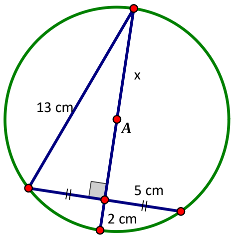
#2) Find $m\widehat{CE}$



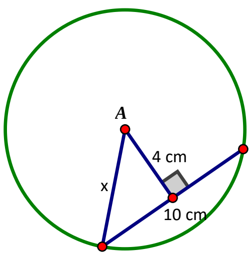
#3) Find x



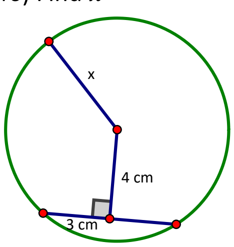
#4) Find x



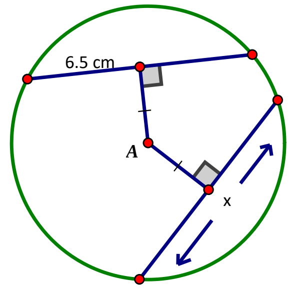
#5) Find x. 2 decimal places.



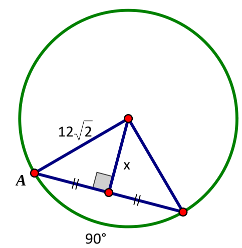
#6) Find x



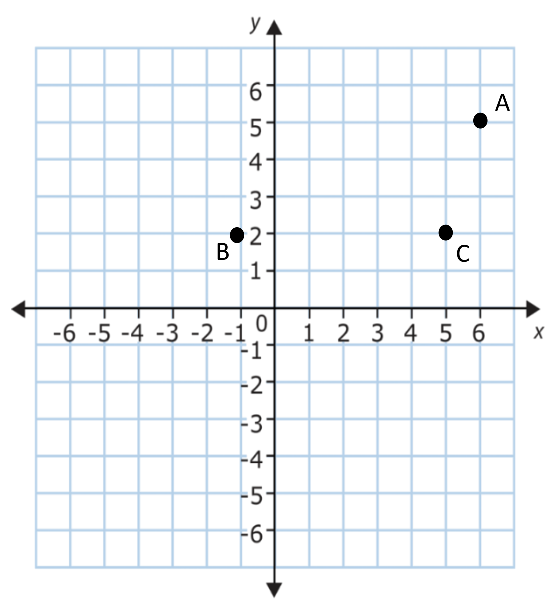
#7) Find x



#8) Find x



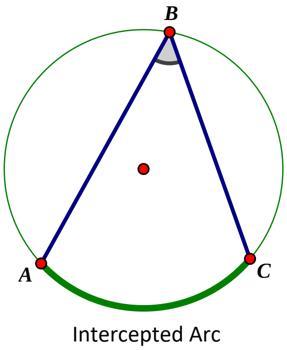
#9) Construct the circle that contains the given points.



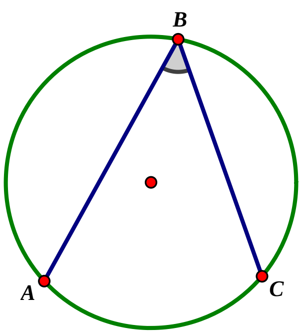
Inscribed Angles

Notes Section 13.3

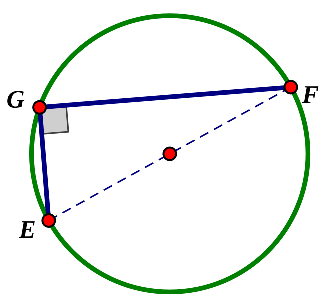
Inscribed Angle: an angle with vertex on the circle and whose sides are chords.



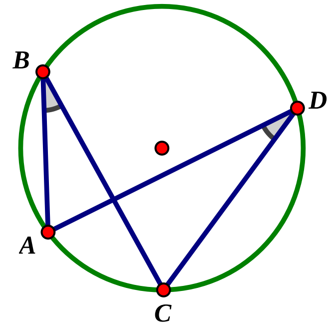
Theorem 13.7: An inscribed angle is half its intercepted arc.



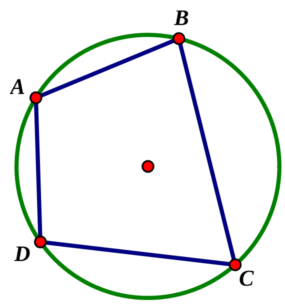
Theorem 13.8: An inscribed angle whose intercepted arc is a semicircle is 90°.



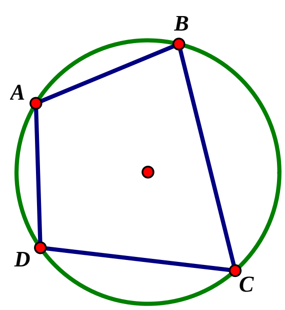
Theorem 13.9: Inscribed angles on the same intercepted arc are congruent.



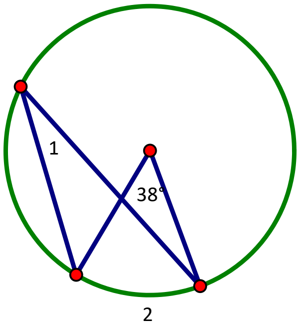
Cyclic Quadrilateral: A quadrilateral that is inscribed in a circle.



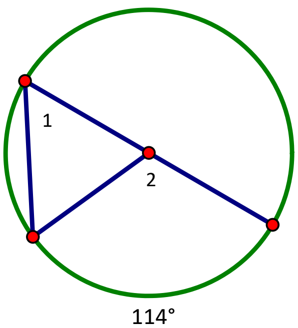
Cyclic Quadrilateral Theorem: Opposite angles in a cyclic quadrilateral are supplementary.



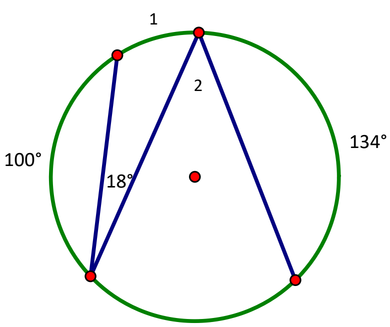
#1) Find $m\angle 1$ and $m\hat{2}$



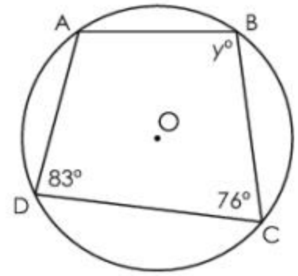
#2) Find $m\angle 1$ and $m\angle 2$



#3) Find $m\angle 2$ and $m\hat{1}$

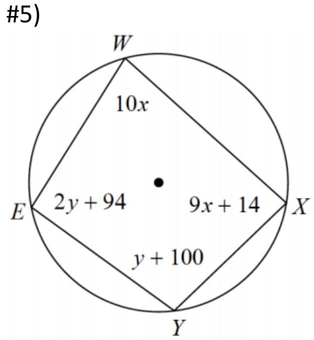


#4)
Quadrilateral ABCD is inscribed in circle O, as shown.

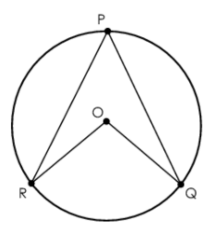


What is the value of y?

#5)



#5)
A teacher draws circle O, ∠RPQ and ∠ROQ, as shown.



The teacher asks students to select the correct claim about the relationship between m∠RPQ and m∠ROQ.

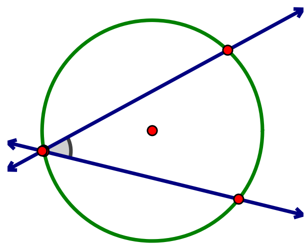
- Claim 1: The measure of ∠RPQ is equal to the measure of ∠ROQ.
- Claim 2: The measure of ∠ROQ is twice the measure of ∠RPQ.

Which claim is correct? Justify your answer.

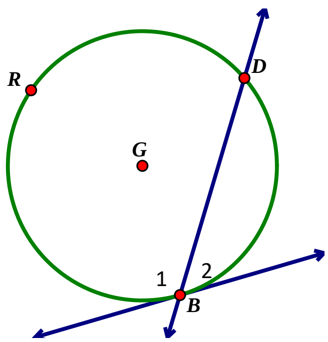
Internal, External & Tangent Angles

Notes Section 13.4

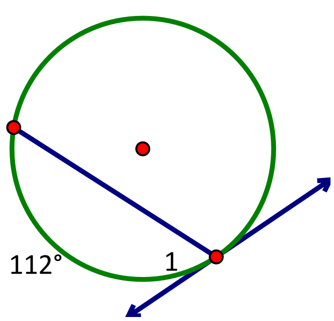
Inscribed Angle (ON)



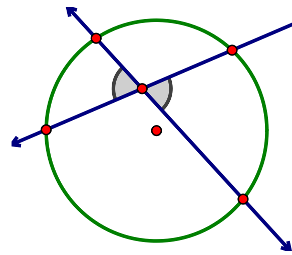
Theorem 13.10: If a tangent and a secant (or chord) intersect at a point on a circle, then the measure of each angle formed is one half the measure of its intercepted arc.



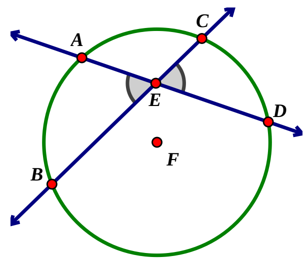
#1) Find m∠1.



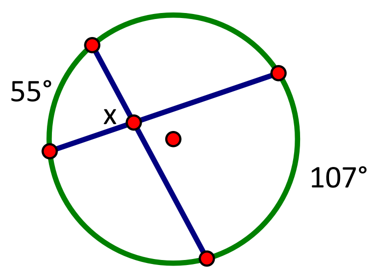
Interior Angle (IN)



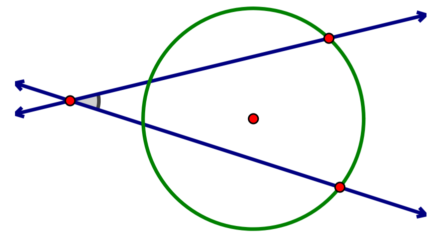
Theorem 13.11: If two secants (or chords) intersect in the interior of a circle, then the measure of each angle formed is one half the sum of the measure of arcs intercepted by the angle and its vertical angle.



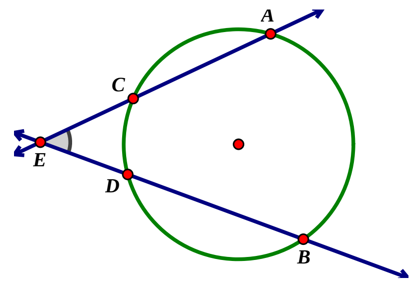
#2) Find *x*.



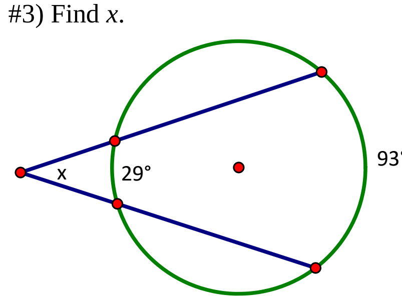
Exterior Angle (OUT)



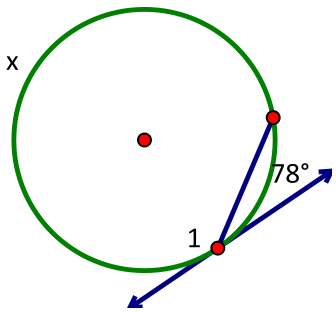
Theorem 13.12: If any combination of secants and tangents intersect in the exterior of a circle, then the measure of each the angle formed is one half the difference of the measure of arcs intercepted arcs.



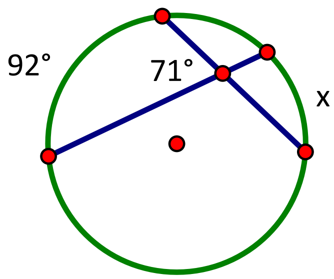
#3) Find *x*.



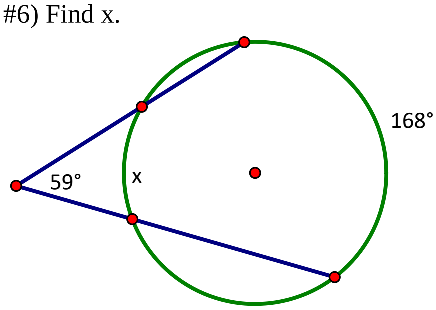
#4) Find x and m∠1.



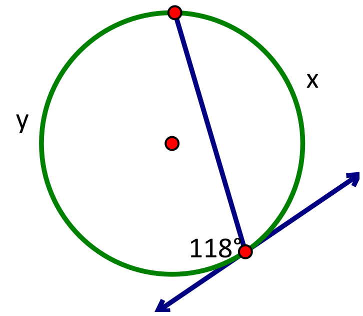
#5) Find x.



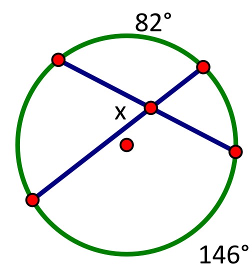
#6) Find x.



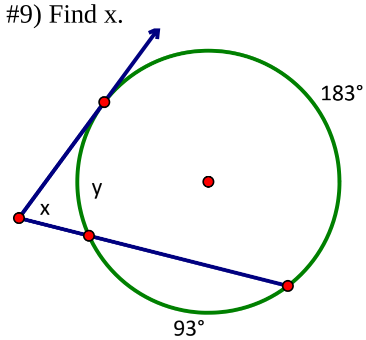
#7) Find x and y.



#8) Find x.



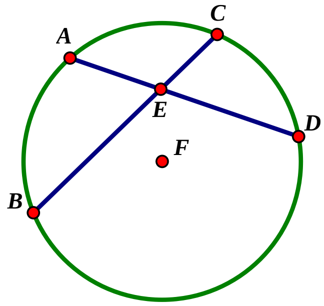
#9) Find x.



Intersecting Chord Properties

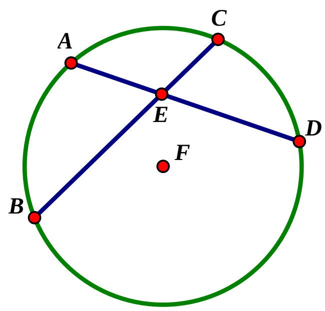
Notes Section 13.5

Theorem 13.13: If two chords intersect in a circle, then the products of the measures of the segments of the chords are equal.

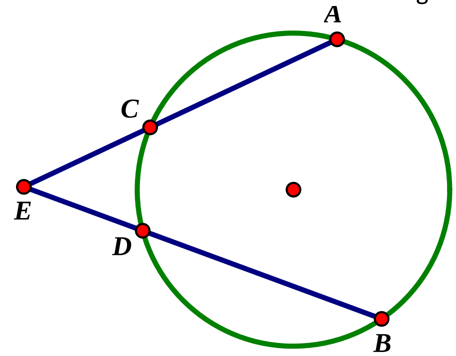


Given: $\overline{AC}$ and $\overline{BD}$ intersect at E.

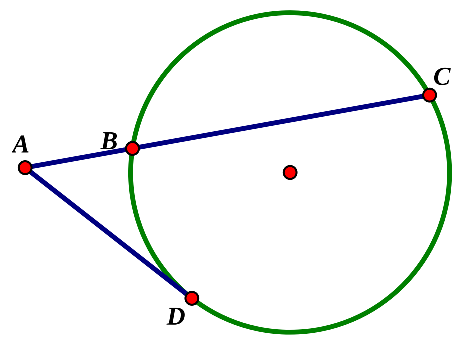
Prove: $AE \cdot ED = CE \cdot EB$



Theorem 13.14: If two secant segments share the same endpoint in the exterior of a circle, then the product of one secant and its external segment is equal to the product of the other secant and its external segment.

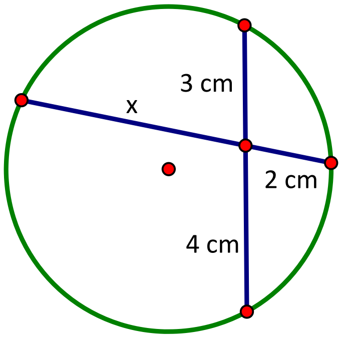


Special Case:

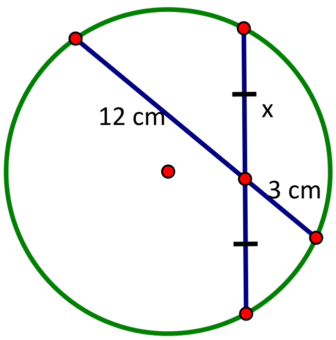


Find the value of x.

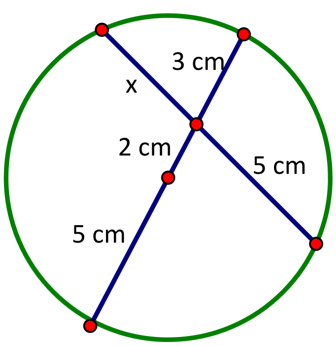
#1)



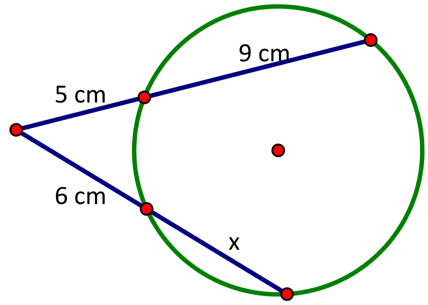
#2)



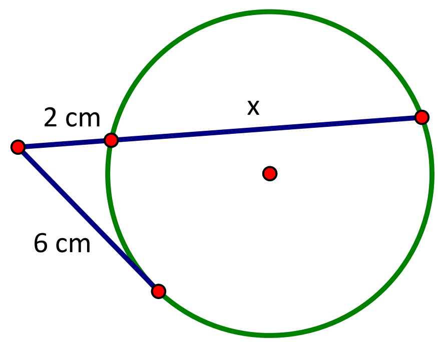
#3)



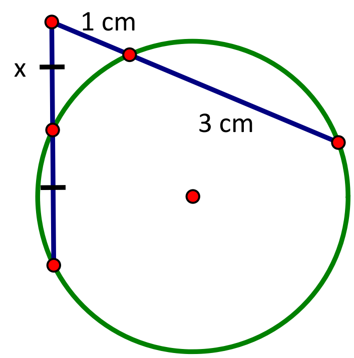
#4)



#5)



#6)



Chapter 13 Summary

1. Summarize the main idea of the chapter

2. Terms (Include name and definition). Also include key example or picture for each term

3. Theorems and Postulates. Also include key example for each theorem or postulate

4. Key examples of the most unique or most difficult problems from notes, homework or application.