$\qquad$

## Chapter 4 - Triangle Congruence

## 4.1

Scalene triangle - A triangle with all three sides having different lengths.

Equilateral triangle - All sides of a triangle are congruent.
Isosceles triangle - A triangle with at least two sides congruent.

- Legs of an isosceles triangle - The congruent sides in an isosceles triangle.
- Vertex angle - The angle formed by the legs in an isosceles triangle.
- Base - The side opposite the vertex angle.
- Base angles - The angles formed by the base.


## Isosceles Triangle Theorem

If two sides of a triangle are congruent, then the angles opposite those sides are congruent.

Corollary 4-1 - A triangle is equilateral if and only if it is equiangular.

Acute triangle - A triangle with all acute angles.
Equiangular triangle - A triangle with all angles congruent.
Obtuse triangle - A triangle with one obtuse angle.

Right triangle - A triangle with one right angle.

- Hypotenuse - The side opposite the right angle in a right triangle.
- Legs of a right triangle - The two sides that form the $90^{\circ}$.


## Converse to the Isosceles Triangle Theorem

If two angles of a triangle are congruent, then the sides opposite those angles are congruent.

Corollary 4-2 - Each angle of an equilateral triangle measures $60^{\circ}$.

Definition of Congruent Triangles (CPCTC) - Two triangles are congruent iff their corresponding parts are congruent.

Terms, Postulates and Theorems

## 4.2

SSS Congruence Postulate (Side-Side-Side)
If the sides of one triangle are congruent to the sides of a second triangle, then the triangles are congruent.

SAS Congruence Postulate (Side-Angle-Side)
If two sides and the included angle of one triangle are congruent to two sides and an included angle of another triangle, then the triangles are congruent.

Median: a segment in a triangle that connects a vertex to the midpoint of the opposite side.

Altitude: a segment in a triangle that connects a vertex to the side opposite forming a perpendicular.

Angle Bisector: a segment that bisects an angle in a triangle and connects a vertex to the opposite side.

Theorem 4.1 - If a median is drawn from the vertex angle of an isosceles triangle, then the median is also an angle bisector and an altitude.

## 4.3

ASA Congruence Postulate (Angle-Side-Angle)
If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, the triangles are congruent.

## AAS Congruence Postulate (Angle-Angle-Side)

If two angles and a nonincluded side of one triangle are congruent to the corresponding two angles and side of a second triangle, the two triangles are congruent.

## 4.4

HL Congruence Theorem (HL) - If the hypotenuse and leg of one right triangle are congruent to the hypotenuse and leg of another right triangle, then the triangles are congruent.
$\qquad$

## Classify by Sides

Scalene triangle - A triangle with all three sides having different lengths.


Equilateral triangle - All sides of a triangle are congruent.


Isosceles triangle - A triangle with at least two sides congruent.

- Legs of an isosceles triangle - The congruent sides in an isosceles triangle.
- Vertex angle - The angle formed by the legs in an isosceles triangle.
- Base - The side opposite the vertex angle.
- Base angles - The angles formed by the base.



## Isosceles Triangle Theorem

If two sides of a triangle are congruent, then the angles opposite those sides are congruent.

Corollary 4-1 - A triangle is equilateral if and only if it is equiangular.

## Classify by Angles

Acute triangle - A triangle with all acute angles.

- Acute angle - An angle greater than $0^{\circ}$ and less than $90^{\circ}$.


Equiangular triangle - A triangle with all angles congruent.


Obtuse triangle - A triangle with one obtuse angle.

- Obtuse angle - An angle more than $90^{\circ}$ and less than $180^{\circ}$.


Right triangle - A triangle with one right angle.

- Right angle - An angle that is $90^{\circ}$.
- Hypotenuse - The side opposite the right angle in a right triangle.
- Legs of a right triangle - The two sides that form



## Converse to the Isosceles Triangle Theorem

If two angles of a triangle are congruent, then the sides opposite those angles are congruent.

Corollary 4-2 - Each angle of an equilateral triangle measures $60^{\circ}$

Definition of Congruent Triangles (CPCTC) - Two triangles are congruent iff their corresponding parts are congruent.


Find the value of $x$.
1.

2.

3.


List pairs of corresponding parts.
4.
$\Delta E F G \cong \Delta Z X Y$


Name congruent figures.
5.

6.
$\Delta R T S \cong \triangle T R G$

$\overline{S R} \cong ?$
7.

$$
\begin{aligned}
& \Delta Z X Y \cong \Delta Z X J \\
& \overline{Y Z} \cong ?
\end{aligned}
$$

8. 

$\Delta L M N \cong \Delta I H N$

$\angle M N L \cong$ ?
$\qquad$

SSS Congruence Postulate (Side-Side-Side)
If the sides of one triangle are congruent to the sides of a second triangle, then the triangles are congruent.

## SAS Congruence Postulate (Side-Angle-Side)

If two sides and the included angle of one triangle are congruent to two sides and an included angle of another triangle, then the triangles are congruent.

Median: a segment in a triangle that connects a vertex to the midpoint of the opposite side.

Altitude: a segment in a triangle that connects a vertex to the side opposite forming a perpendicular.

Angle Bisector: a segment that bisects an angle in a triangle and connects a vertex to the opposite side.

Theorem 4.1 - If a median is drawn from the vertex angle of an isosceles triangle, then the median is also an angle bisector and an altitude.

State if the two triangles are congruent. If they are, state why.
1.

2.

3.

4.

5.

6.


Given: $\boldsymbol{H}$ is the midpoint of $\overline{G T}$ $\overline{H R} \cong \overline{I H}$

Prove: $\Delta G H I \cong \triangle T H R$


WHY ARE THE TWO TRIANGLES CONGRUENT?

| STATEMENTS | REASONS |
| :--- | :--- |
| $1 . \overline{H R} \cong \overline{I H}$ <br> $H$ is the midpoint of $\overline{G T}$ | 1. |
| 2. | 2. |
| 3. | 3. |
| 4. | 4. |

Given: $\triangle A C B$ is an isosceles triangle with base $\overline{A B}$ $\overline{C P}$ is an angle bisector of $\angle A C B$

Prove: $\triangle A C P \cong \triangle B C P$


WHY ARE THE TWO TRIANGLES CONGRUENT? $\qquad$

| STATEMENTS | REASONS |
| :--- | :--- |
| $1 . \Delta A C B$ is an isosceles triangle | 1. |
| 2. | 2. |
| 3. | 3. |
| 4. | 4. |
| 5. | 5. |

$\qquad$

## AAS and ASA

ASA Congruence Postulate (Angle-Side-Angle)
If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, the triangles are congruent.

## AAS Congruence Postulate (Angle-Angle-Side)

If two angles and a nonincluded side of one triangle are congruent to the corresponding two angles and side of a second triangle, the two triangles are congruent.

State if the two triangles are congruent. If they are, state why.
1.

2.

3.

4.

5.

6.

7.

8.

9.


$$
\text { Given: } \frac{\angle X}{\overline{X J} \cong \overline{I H}}
$$

Prove: $\Delta J H I \cong \Delta H J X$


WHY ARE THE TWO TRIANGLES CONGRUENT?

| STATEMENTS | REASONS |
| :--- | :--- |
| $1 . \overline{X J} \\| \overline{I H}$ |  |
| $\angle X \cong \angle I$ |  |$) 1 . \quad 2 . \quad 3$.

Given: $\overline{A C} \cong \overline{B C}$ $\overline{C P}$ is perpendicular to $\overline{A B}$

Prove: $\triangle A C P \cong \triangle B C P$


WHY ARE THE TWO TRIANGLES CONGRUENT?

| STATEMENTS | REASONS |
| :--- | :--- |
| $1 . \overline{A C} \cong \overline{B C}$ |  |
| $\overline{C P}$ is perpendicular to $\overline{A B}$ | 1. |
| 2. | 2. |
| 3. | 3. |
| 4. | 4. |
| 5. | 5. |
| 6. | 6. |

$\qquad$
HL
HL Congruence Theorem (HL) - If the hypotenuse and leg of one right triangle are congruent to the hypotenuse and leg of another right triangle, then the triangles are congruent.

Given: $\overline{X J} \cong \overline{H I}$
$\overline{X J} \| \overline{I H}$

Prove: $\overline{X H} \cong \bar{J}$


WHY ARE THE TWO TRIANGLES CONGRUENT? $\qquad$

| STATEMENTS | REASONS |
| :--- | :--- |
| $1 . \overline{X J} \\| \overline{I H}$ |  |
| $\overline{X J} \cong \overline{H I}$ | 1. |
| 2. | 2. |
| 3. | 3. |
| 4. | 4. |
| 5. | 5. |

Given: $\angle H Y P$ and $\angle L E G$ are right angles $\overline{H Y} \cong \overline{E L}$
$\overline{\boldsymbol{H P}} \cong \overline{\boldsymbol{L} \boldsymbol{G}}$

Prove: $\triangle H Y P \cong \triangle L E G$


## WHY ARE THE TWO TRIANGLES CONGRUENT?

$\qquad$

| STATEMENTS | REASONS |  |
| :--- | :--- | :--- |
| 1. | $\angle H Y P$ and $\angle L E G$ are right angles | 1. |
| $\overline{H Y} \cong \overline{E L}$ |  |  |
| 2. | 2. |  |
| 3. | 3. |  |


| ALGEBRA REVIEW |  |  |  |
| :---: | :---: | :---: | :---: |
| SOLVE $26=-7+3(2 x-4)-x$ | $y=-\frac{3}{2} x$ |  | $\begin{gathered} \text { MULTIPLY } \\ (2 x-1)(2 x+1) \end{gathered}$ |
| $\begin{aligned} & \text { SOLVE } \\ & \frac{5}{12}=\frac{-8}{x} \end{aligned}$ | $y=1-x$ | GRAPH | $\begin{gathered} \text { FACTOR } \\ x^{2}+23 x+42 \end{gathered}$ |

## Chapter 4 Summary

1. Summarize the main idea of the chapter
2. Terms (Include name and definition). Also include key example or picture for each term
3. Theorems and Postulates. Also include key example for each theorem or postulate
4. Key examples of the most unique or most difficult problems from notes, homework or application.
$\qquad$

## Bisectors, Medians and Altitudes

Median: a segment in a triangle that connects a vertex to the midpoint of the opposite side.


Altitude: a segment in a triangle that connects a vertex to the side opposite forming a perpendicular.


Angle Bisector: a segment that bisects an angle in a triangle and connects a vertex to the opposite side.


Perpendicular Bisector: a segment in a triangle that passes through the midpoint of a side and is perpendicular to that side.


Theorem 5-1.2: A point is on the perpendicular bisector IFF it is equidistant from the endpoints of the segment.


Notes Section 5.1
Draw and label a figure to illustrate each situation. \#1) $\overline{P T}$ and $\overline{R S}$ are medians of triangle $\triangle \mathrm{PQR}$ and intersect at V .
\#2) $\overline{A D}$ is a median and an altitude of $\triangle A B C$.
\#3) $\triangle \mathrm{DEF}$ is a right triangle with right angle at $\mathrm{F} . \overline{F G}$ is a median of $\triangle$ DEF and $\overline{G H}$ is the perpendicular bisector of $\overline{D E}$.

State whether each sentence is always, sometimes, or never true.
\#4) Three medians of a triangle intersect at a point inside the triangle.
\#5) The three angle bisectors of a triangle intersect at a point outside the triangle.
\#6) The three altitudes of a triangle intersect at a vertex of the triangle.


B

\#9) Find the midpoint of $A(2,4)$ and $B(-5,8)$
\#10) Find $m \angle A B C$ if $\overline{B D}$ is an angle bisector of $\triangle A B C$.

\#11) $\overline{A D}$ is a perpendicular bisector of $\overline{B C}$. Find x and y .


B

## Chapter 6 - Quadrilaterals

Section 6.2
Parallelogram: a quadrilateral with both pairs of opposite sides parallel.

Theorem 6-1: Opposite sides of a parallelogram are congruent.
Theorem 6-2: Opposite angles of a parallelogram are congruent.

Theorem 6-3: Consecutive angles in a parallelogram are supplementary.

Theorem 6-4: If a parallelogram has one right angle then it has four right angles.

Theorem 6-5: The diagonals of a parallelogram bisect each other.

Theorem 6-6: Each diagonal of a parallelogram separates the parallelogram into two congruent triangles.

Section 6.3
Theorem 6-7: If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

Theorem 6-8: If one pair of opposite sides of a quadrilateral is both parallel and congruent, then the quadrilateral is a parallelogram.

Theorem 6-9: If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

Theorem 6-10: If both pairs of opposite angles in a quadrilateral are congruent, then the quadrilateral is a parallelogram.

Section 6.4
Rectangle: a quadrilateral with four right angles. (Also could define as a parallelogram with one right angle.)

Theorem 6-11.12: A parallelogram is a rectangle IFF its diagonals are congruent.

Terms, Theorems \& Postulates
Section 6.5
Rhombus: A quadrilateral with four congruent sides. (Also could be defined as a parallelogram with four congruent sides.)

Theorem 6-13.14: A parallelogram is a rhombus IFF its diagonals are perpendicular.

Theorem 6-15: Each diagonal of a rhombus bisects a pair of opposite angles.

Square:
(a rectangular rhombus; a rhombicular rectangle.) A quadrilateral that is both a rhombus and a rectangle.

Section 6.6
Trapezoid: a quadrilateral with exactly one pair of parallel sides.
Bases: the parallel sides of a trapezoid.
Legs: $\quad$ the nonparallel sides of a trapezoid.
Pair of base angles: two angles in a trapezoid that share a common base.

Isosceles trapezoid: a trapezoid with congruent legs.

Theorem 6-16:
Both pairs of base angles of an isosceles trapezoid are congruent.

Theorem 6-17:
The diagonals of an isosceles trapezoid are congruent.

## Median of a Trapezoid:

a segment that connects the midpoints of the legs.

## Theorem 6-18:

The median of a trapezoid is parallel to the bases and its measure is one half the sum of the measures of the bases.

## Solving Systems of Equations

Notes Section 6.1

Solve each system of equations by substitution or elimination. If the system does not have exactly one solution, state whether it has no solution or infinitely many solutions.
\#1)
$x=7$
$5 y+x=12$
\#2) $5 x+4 y=-9$
$2 x-4 y=-40$
\#3)
$y=3 x-2$
$3 x-y=7$

|  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| \# |  |  | 苍 |  |  | \# |
|  |  |  |  |  |  |  |

$\qquad$

## Parallelograms

Parallelogram: a quadrilateral with both pairs of opposite sides parallel.


Theorem 6-1: Opposite sides of a parallelogram are congruent.


Theorem 6-2: Opposite angles of a parallelogram are congruent.


Theorem 6-3: Consecutive angles in a parallelogram are supplementary.


Theorem 6-4: If a parallelogram has one right angle then it has four right angles.


Theorem 6-5: The diagonals of a parallelogram bisect each other.


Theorem 6-6: Each diagonal of a parallelogram separates the parallelogram into two congruent triangles.


Is each quadrilateral a parallelogram? Justify your answer.
\#1)

\#2)


If each quadrilateral is a parallelogram, find the value of $x$, $y$, and $z$.
\#3)

\#4)


With the given information, answer each question. \#5) Given parallelogram PQRS with $\mathrm{m} \angle \mathrm{P}=2 \mathrm{y}$ and $\mathrm{m} \angle \mathrm{Q}=4 \mathrm{y}+30$, find the $\mathrm{m} \angle \mathrm{R}$ and $\mathrm{m} \angle \mathrm{S}$.
\#6) If NCTM is a parallelogram, $m \angle N=12 x+10 y+5$, $m \angle C=9 x$, and $m \angle T=6 x+15 y$, find $m \angle M$.

## Tests for Parallelograms

Theorem 6-7:
If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.


Theorem 6-8:
If one pair of opposite sides of a quadrilateral is both parallel and congruent, then the quadrilateral is a parallelogram.


Theorem 6-9:
If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.


Theorem 6-10:
If both pairs of opposite angles in a quadrilateral are congruent, then the quadrilateral is a parallelogram.


Notes Section 6.3
Determine if each quadrilateral must be a parallelogram. Justify your answer.
\#1)

\#2)

\#3)


Use parallelogram ABCD and the given information to find each value.
\#4) $m \angle A B C=50^{\circ}$. Find $m \angle B C D$

\#5) $A B=11, B C=2, m \angle A D C=84^{\circ}$. Find $D C$.

\#6) What values must $x$ and $y$ be in order for quadrilateral to be a parallelogram? $\mathrm{ST}=\mathrm{x}+3 \mathrm{y}, \mathrm{TA}=6, \mathrm{PT}=4 \mathrm{x}+2 \mathrm{y}$ and TN = 14

\#7) The coordinates of the vertices of quadrilateral ABCD are $A(-1,3), B(2,1), C(9,2)$, and $D(6,4)$. Determine if the quadrilateral $A B C D$ is a parallelogram.

Option 1: Use the distance formula to find the length of all four sides.
*If odposite lengths are the same. then the auad is a parallelogram.
Option 2: Use the slope formula to find the slope of all four sides.
*If opposite slopes are the same, then the quad is a parallelogram.
Option 3: Find the slopes and lengths of one pair of opposite sides. *If the pair of opposite sides have the same slope and length, then the quad is a parallelogram.

Option 4: Find the midpoints of the diagonals.
*If the midpoints of the diagonals are the same, then the quad is a parallelogram.
$A(-1,3), B(2,1), C(9,2)$, and $D(6,4)$.

## Rectangles

Notes Section 6.4
Rectangle: a quadrilateral with four right angles. (Also could define as a parallelogram with one right angle.)


Theorem 6-11.12: A parallelogram is a rectangle IFF its diagonals are congruent.


Use rectangle MATH and given information to solve each problem.
\#1) $\mathrm{HP}=10$. Find MT .

\#2) $m \angle 1=40^{\circ}$. Find $m \angle 2$


Draw a counterexample to show that each statement below is false.
\#3) If a quadrilateral has one pair of congruent sides, it is a rectangle.
\#4) If a quadrilateral has two pairs of congruent sides, it is a rectangle.

Find the values of $x$ and $y$ in rectangle PQRS.
\#5) $T R=3 x-12 y, T Q=-2 x+9 y+4, S T=3$


Determine whether $A B C D$ is a rectangle. Explain.
\#6) $A(1,2), B(3,6), C(9,3), D(7,-1)$

Option 1: Use the distance formula to find the length of all four sides.
Use the slope formula to find the slopes of two consecutive sides.
*If opposite lengths are the same, and consecutive slopes are perpendicular, then the quad is a rectangle.

Option 2: Use the slope formula to find the slope of all four sides. *If opposite slopes are parallel and consecutive slopes are perpendicular, then the quad is a rectangle.

Option 3: Find the midpoints of the diagonals.
Find the lengths of the diagonals.
*If the midpoints of the diagonals are the same and the diagonals are the same length, then the quad is a rectangle.
$\qquad$

## Squares and Rhombi

Rhombus:
A quadrilateral with four congruent sides. (Also could be defined as a parallelogram with four congruent sides.)


Theorem 6-13.14:
A parallelogram is a rhombus IFF its diagonals are perpendicular.


Theorem 6-15:
Each diagonal of a rhombus bisects a pair of opposite angles.


Square:
(a rectangular rhombus; a rhombicular rectangle.) A quadrilateral that is both a rhombus and a rectangle.


Name all the quadrilaterals - parallelogram, rectangle, rhombus, or square - that have each property. \#1) The opposite sides are parallel.
\#2) The opposite sides are congruent.
\#3) All sides are congruent.
\#4) It is equiangular and equilateral.

Use rhombus BEAC with $\mathrm{BA}=10$ to determine whether each statement is true or false. Justify your answer.
\#5) $\mathrm{CE}=10$
\#6) $\overline{C E} \perp \overline{A B}$


Use rhombus IJKL and the given information to solve each problem.
\#7) If $\mathrm{m} \angle 3=4(x+1)$ and $\mathrm{m} \angle 5=2(x+1)$, find $x$.


Determine whether EFGH is a parallelogram, rectangle, rhombus, or square. List all that apply.
\#8) $\mathrm{E}(6,5), F(2,3), G(-2,5), H(2,7)$

> To determine if a quad is a parallelogram.
> The diagonals must have the same midpoint.
> To determine if a quad is a rectangle.
> The midpoints of the diagonals must be the same and the diagonals must have the same length.

To determine if a quad is a rhombus.
The midpoints of the diagonals must be the same and the diagonals must be perpendicular

To determine if a quad is a square. The quad must be a rectangle and a rhombus.

## Trapezoids

Notes Section 6.6

Trapezoid: a quadrilateral with exactly one pair of parallel sides.

Bases: the parallel sides of a trapezoid.

Legs: the nonparallel sides of a trapezoid.


Pair of base angles: two angles in a trapezoid that share a common base.


Isosceles trapezoid: a trapezoid with congruent legs.


Theorem 6-16:
Both pairs of base angles of an isosceles trapezoid are congruent.


Theorem 6-17:
The diagonals of an isosceles trapezoid are congruent.


Median of a Trapezoid:
a segment that connects the midpoints of the legs.


Theorem 6-18:
The median of a trapezoid is parallel to the bases and its measure is one half the sum of the measures of the bases.


If possible, draw a trapezoid that has the following characteristics. If the trapezoid cannot be drawn, explain why.
\#1) Four congruent sides.
\#2) One right angle.
\#3) One pair of opposite angles congruent.
\#4) Congruent diagonals.

PQRS is an isosceles trapezoid with bases $\overline{P S}$ and $\overline{Q R}$. Use the figure and the given information to solve each problem.
\#5) If $T V=2 x+5$ and $P S+Q R=5 x+3$, find $x$.

\#6) If the measure of the median of an isosceles trapezoid is 7.5 , what are the possible integral measures for the bases?
\#7) $\overline{U R}$ is the median of a trapezoid with bases $\overline{O N}$ and $\overline{T S}$. If the coordinates of the points are $U(2$, 2), $R(6,2), O(6,-2), N(0,-2)$, find the coordinates of $T$ and $S$.


## Chapter 6 Summary

1. Summarize the main idea of the chapter
2. Terms (Include name and definition). Also include key example or picture for each term
3. Theorems and Postulates. Also include key example for each theorem or postulate
4. Key examples of the most unique or most difficult problems from notes, homework or application.

## Chapter 7 - Similarity

Section 7.1
Ratio: a comparison of two quantities.

Proportion: an equation stating that two ratios are equal.

## Section 7.2

Rate: a ratio of two measurements that may have different types of units.

Similar Polygons: Two polygons are similar IFF their corresponding angles are congruent and the measures of their corresponding sides are proportional.

Scale Factor: The ratio of the lengths of two corresponding sides of two similar polygons

Section 7.3
AA Similarity: If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.

SSS Similarity: If the measures of the corresponding sides of two triangles are proportional, then the triangles are similar.

SAS Similarity: If the measures of two sides of a triangle are proportional to the measures of two corresponding sides of another triangle and the included angles are congruent, then the triangles are similar.

Theorem 7-3: Similarity of triangles is reflexive, symmetric, and transitive.

Terms, Theorems \& Postulates

## Section 7.4

Triangle Proportionality: A line, that intersects two sides of a triangle in two distinct points, is parallel to the third side IFF it separates these sides into segments of proportional lengths.

Theorem 7-6: a segment whose endpoints are the midpoints of two sides of a triangle is parallel to the third side of the triangle and its length is one-half the length of the third side.

Corollary 7-1: If three or more parallel lines intersect two transversals, then they cut off the transversals proportionally.

Corollary 7-2: If three or more parallel lines cut off congruent segments on one transversal then they cut off congruent segments on every transversal.

Section 7.5
Proportional Perimeter: If two triangles are similar, then the perimeters are proportional to the measures of corresponding sides.

Proportional Altitudes Theorem: If two triangles are similar, then the measures of the corresponding altitudes are proportional to the measures of the corresponding sides.

Proportional Angle Bisectors Theorem: If two triangles are similar, then the measures of the corresponding angle bisectors are proportional to the measures of the corresponding sides.

Proportional Medians Theorem If two triangles are similar, then the measures of the corresponding medians are proportional to the measures of the corresponding sides.

Angle Bisector Theorem: An angle bisector in a triangle separates the opposite side into segments that have the same ratio as the other two sides.

The quiz will consist of one matching section and one multiple choice section. The matching section will contain all terms and the theorems that have names.
The multiple choice section will contain all theorems, postulates, and corollaries that have no names. I will remove a word from the sentence and give you three or four choices to complete the sentence.

## Properties of Proportions

Ratio: a comparison of two quantities.

What is the ratio of female students to male students in this class?

What is the ratio to Twinkie riders to car riders in this class?

Proportion: an equation stating that two ratios are equal.

Example:

Solve each proportion.
\#1)

$$
\frac{x}{12}=\frac{8}{30}
$$

Notes Section 7.1

$$
\frac{x+6}{10}=\frac{2 x-5}{3}
$$

\#4)

$$
\frac{10}{9}=\frac{30}{x+2}
$$

\#2)
$\frac{7-x}{9}=\frac{2}{6}$
\#5) On a bike, the ratio of the number of rear sprocket teeth to the number of front sprocket teeth is equivalent to the number of rear sprocket wheel revolutions to the number of pedal revolutions. If there are 8 rear sprocket teeth and 18 front sprocket teeth, how many revolutions of the rear sprocket wheel will occur for 5 revolutions of the pedal?
\#6) One way to determine the strength of a bank is to calculate its capital-to-assets ratio as a percent. A weak bank has a ratio of less than 4\%. The Gnaden National Bank has a capital of $\$ 177,000$ and assets of $\$ 4,450,000$. Is it a weak bank? Explain.
\#7) The ratio of the measures of the angles of a triangle is $3: 5: 7$. What is the measure of each angle in the triangle?
\#8) On a map of Ohio, three fourths of an inch represents 15 miles. If it is approximately 10 inches from Sandusky to Cambridge on the map, what is the actual distance in miles?

## Similar Polygons

Similar Polygons: Two polygons are similar IFF their corresponding angles are congruent and the measures of their corresponding sides are proportional.

Scale Factor: The ratio of the lengths of two corresponding sides of two similar polygons

Determine whether each pair of figures is similar. Justify your answer.
\#1)


Draw and label a pair of polygons for each. If it is impossible to draw two such figures, write "Mission: Impossible."
\#3) two pentagons that are similar
\#4) two squares that are not similar

Given two similar polygons find the value of $x$ and $y$.
\#5)


Make a scale drawing using the given scale. \#8) A basketball court is 84 feet by 50 feet. Scale: $\frac{1}{8}$ inch $=2 \mathrm{ft}$.

IF quadrilateral PQRS is similar to $A B C D$, find the scale factor of quadrilateral PQRS to quadrilateral $A B C D$.


## Similar Triangles

Notes Section 7.3
AA Similarity: If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.

Theorem 7-3: Similarity of triangles is reflexive, symmetric, and transitive.

SSS Similarity: If the measures of the corresponding sides of two triangles are proportional, then the triangles are similar.

Reflexive

Symmetric

Transitive

SAS Similarity: If the measures of two sides of a triangle are proportional to the measures of two corresponding sides of another triangle and the included angles are congruent, then the triangles are similar.
\#1) Determine if each pair of triangles is similar. If similar, state the reason and find the missing measure.

\#2) In the figure, $\overline{S T} / / \overline{P R}, \mathrm{QS}=3, \mathrm{SP}=1$, and $\mathrm{TR}=1.2$. Find QT.

\#3) If $T S=6, Q P=4, R S=x+1$, and $Q R=3 x-4$, find the value of $x$

\#4) Identify the similar triangles in each figure. Explain your answer.


## Parallel Lines \& Proportional Parts

Triangle Proportionality: A line, that intersects two sides of a triangle in two distinct points, is parallel to the third side IFF it separates these sides into segments of proportional lengths.


Midsegment: A segment in a triangle with endpoints that are the midpoints of two sides of the triangle.


Theorem 7-6: A midsegment is parallel to the third side of the triangle and its length is one-half the length of the third side.


Corollary 7-1: If three or more parallel lines intersect two transversals, then they cut off the transversals proportionally.


Corollary 7-2: If three or more parallel lines cut off congruent segments on one transversal then they cut off congruent segments on every transversal.


Notes Section 7.4
\#1) Find the value of $x$.

\#2) Determine if $\overline{B D} / / \overline{A E}$.
$C A=15, A B=3, C D=8, C E=10$
C

B


A
E
\#3) Find the value of $x$.

\#4) Find the value of $x$.


\#6) Find the value of $x$.


## Parts of Similar Triangles

Proportional Perimeter Theorem: If two triangles are similar, then the perimeters are proportional to the measures of corresponding sides.


Proportional Altitudes Theorem: If two triangles are similar, then the measures of the corresponding altitudes are proportional to the measures of the corresponding sides.


Proportional Angle Bisectors Theorem: If two triangles are similar, then the measures of the corresponding angle bisectors are proportional to the measures of the corresponding sides.


Proportional Medians Theorem: If two triangles are similar, then the measures of the corresponding medians are proportional to the measures of the corresponding sides.


Angle Bisector Theorem: An angle bisector in a triangle separates the opposite side into segments that have the same ratio as the other two sides.

\#1) Find the value of $x$.

\#2) Find the value of $x$.

\#3) $\triangle A B C$ is similar to $\triangle X Y Z$. Segments $\overline{A K}$ and $\overline{Q X}$ are medians of the triangles.
$A K=4, B K=3, Y Z=x+2, Q X=2 x-5$. Find $Q Z$.

$\triangle A B C$ is similar to $\triangle X Y Z$. Determine if each proportion is true or false.

\#4) $\frac{A B}{X Y}=\frac{A C}{X Z}$
\#5) $\frac{A K}{B C}=\frac{X Q}{Y Z}$
$\begin{array}{ll}\text { \#6) } \frac{B C}{Y Z}=\frac{X Y}{A B} & \text { \#7) } \frac{A B}{A K}=\frac{X Y}{X Q}\end{array}$

## Chapter 7 Summary

1. Summarize the main idea of the chapter
2. Terms (Include name and definition). Also include key example or picture for each term
3. Theorems and Postulates. Also include key example for each theorem or postulate
4. Key examples of the most unique or most difficult problems from notes, homework or application.

## Transformations - Isometries <br> Notes T. 1 (G.CO.A.2)

An ISOMETRIC TRANSFORMATION (RIGID MOTION) is a transformation that
$\qquad$
$\qquad$

Synonym for isometry $\qquad$

## Isometric Transformations

Rotations, Translations, \& Reflections

This is a $\qquad$



This is a $\qquad$


This is a $\qquad$



A NON-ISOMETRIC TRANSFORMATION (NON-RIGID MOTION) is a transformation
$\qquad$

Non-Isometric Transformations
Dilations and Stretches

This is a $\qquad$ which produces $\qquad$ figures.


This is a $\qquad$


This is also a $\qquad$


1. Circle which of the following are isometric transformations? (there may be more than 1 answer) And determine which transformation took place by writing reflection, translation, rotation, dilation, stretch or other under each image.

$\qquad$
$\qquad$
2. Circle which of the following are isometric transformations? (there may be more than 1 answer) And determine which transformation took place by writing reflection, translation, rotation, dilation, stretch or other under each image.


|  | Transformation |
| :---: | :---: |
| a) Pre-Image Points | Coordinate Rule |
| A (-1,1) | $(x, y) \rightarrow(-y, x)$ |
| B $(0,4)$ | Image Points |
| C $(4,1)$ <br> Isometry? Yes or No | $A^{\prime}$ |
|  | $B^{\prime}$ (___ ${ }^{\text {_ }}$ ) |
| Transformation Type: | $C^{\prime}(\ldots, \quad, \quad)$ |

4. Determine the coordinates of the image, plot the image and determine if it is an isometric transformation.


|  | Transformation |
| :---: | :---: |
| a) Pre-Image Points | Coordinate Rule |
| A (0,0) | $(x, y) \rightarrow(x,-2 y)$ |
| B $(1,3)$ | Image Points |
| C $(5,0)$ <br> Isometry? Yes or No | $A^{\prime}(\ldots, \ldots)$ |
|  | $B^{\prime}$ (___ ${ }^{\text {_ }}$ ) |
| Transformation Type: | $C^{\prime}$ (___ ${ }^{\text {_ }}$ ) |

$\qquad$

## Transformations - Symmetry

What does it mean to carry a shape onto itself?

What types of symmetry are there?

LINE SYMMETRY (or REFLECTIONAL SYMMETRY) What is the definition of Line Symmetry?

How many lines of symmetry does each figure have?


## ROTATIONAL SYMMETRY

A geometric figure has rotational symmetry if the figure is the image of itself under a rotation about a point through any angle whose measure is strictly between $0^{\circ}$ and $360^{\circ} .0^{\circ}$ and $360^{\circ}$ are excluded from counting as having rotational symmetry because it represents the starting position.

ANGLE OF ROTATION - When a shape has rotational symmetry we sometimes want to know what the angle of rotational symmetry is. To determine this we determine the SMALLEST angle through which the figure can be rotated to coincide with itself. This number will always be a factor of $360^{\circ}$.

ORDER OF ROTATION SYMMETRY -- The number of positions in which the object looks exactly the same is called the order of the symmetry. When determining order, the last rotation returns the object to its original position.
Order 1 implies no true rotational symmetry since a full 360 degree rotation was needed.

Determine the angle or rotation and order of rotation.

Angle = $\qquad$ ${ }^{\circ}$
Order $=$ $\qquad$

Angle = $\qquad$
 ${ }^{\circ}$
Angle = $\qquad$ - Angle $=$ _-
Order = $\qquad$ Order $=$ $\qquad$ Order $=$ $\qquad$

Angle $=$ $\qquad$ ${ }^{\circ}$

$\qquad$

Angle = $\qquad$ - Angle $=$ _-
Order = $\qquad$ Order = $\qquad$ Order = $\qquad$ Order = $\qquad$

Shade each figure so it has the indicated angle or rotation and order of rotation.


## POINT SYMMETRY

Point Symmetry exists when a figure is built around a point such that every point in the figure has a matching point that is the SAME DISTANCE from the central point but IN THE OPPOSITE DIRECTION.

A simple test to determine whether a figure has point symmetry is to turn it upside-down and see if it looks the same. A figure that has point symmetry is unchanged in appearance by a 180 degree rotation.

You will notice that the point of rotation is a midpoint between every point and its image.

$\qquad$

## Transformations - Isometries

REFLECTION DEFINITION
A reflection in a line $m$ is a isometric transformation that maps every point $P$ in the plane to a point $P^{\prime}$, so that the following properties are true:

1. If point $P$ is NOT on line $m$, then line $m$ is the perpendicular bisector of $\overline{P P^{\prime}}$.
2. If point P is $\mathbf{O N}$ line $m$, then $\mathrm{P}=\mathrm{P}^{\prime}$

The line of reflection is the $\qquad$ of the segment joining every point and its image.

$$
r_{m}(\Delta A B C)=\Delta A^{\prime} B^{\prime} C^{\prime}
$$

## CHARACTERISTICS

DISTANCES FROM PRE-IMAGE TO IMAGE Points in the plane move $\qquad$ distances, depending on their distance from the line of reflection. Points farther away from the line of reflection move a $\qquad$ distance than those closer to the line of reflection. Notice how $\overline{A A^{\prime}} / / \overline{B B^{\prime}} / / \overline{C C^{\prime}}$.

## ORIENTATION

The pre-image has $\qquad$ orientation to its image. The reflection creates a $\qquad$ image.

## SPECIAL POINTS

The points on the line of reflection $\qquad$ .

## TRANSLATION DEFINITION

A translation is an isometric transformation that maps every two points $A$ and $B$ in the plane to points $A^{\prime}$ and $B^{\prime}$, so that the following properties are true:

1. $A A^{\prime}=B B^{\prime}$ (a fixed distance).
2. $\overline{A A^{\prime}} \| \overline{B B^{\prime}}$ (a fixed direction).

$$
T_{\langle x, y\rangle}(\Delta A B C)=\Delta A^{\prime} B^{\prime} C^{\prime}
$$

## CHARACTERISTICS

 DISTANCES FROM PRE-IMAGE TO IMAGEPoints in the plane all map the $\qquad$

## ORIENTATION

The pre-image has $\qquad$ orientation as its image.

SPECIAL POINTS
There are $\qquad$ special points

## SPECIAL TRANSLATION PROPERTY -

## TRANSLATING AN ANGLE ALONG ONE OF ITS RAYS

A translation of $\angle \mathrm{ABC}$ by vector $\overrightarrow{\boldsymbol{B A}}$ maps all points so:

1. $\angle A B C \cong \angle A^{\prime} B^{\prime} C^{\prime}$ (Isometry)
2. $B, A, B^{\prime}$ and $A^{\prime}$ are collinear (translation on angle ray)

Because the two angles are equal and formed on the same ray, then:

$$
\overrightarrow{B C} \| \overrightarrow{B^{\prime} C^{\prime}}
$$

All segments that are translated are parallel to each other.

## ROTATION DEFINITION

A rotation about a Point $O$ through $\Theta$ degrees is an isometric transformation that maps every point $P$ in the plane to a point $P^{\prime}$, so that the following properties are true:

1. If point $P$ is NOT point $O$, then $O P=O P$ ' and $\mathrm{m} \angle \mathrm{POP}^{\prime}=\Theta^{\circ}$.
2. If point $P$ IS the point of rotation, then $P=P$ '. The center of rotation is the ONLY point in the plane that is unaffected by a rotation.

A rotation is an isometric transformation that turns a figure about a fixed point called the center of rotation (notation $\mathrm{R}_{\text {center, degree }}$ ).

An object and its rotation are the same shape and size, but the figures may be turned in different directions.

$$
R_{o, \theta}(\Delta A B C)=\Delta A^{\prime} B^{\prime} C^{\prime}
$$



## ROTATION DIRECTION



## CHARACTERISTICS

DISTANCES FROM PRE-IMAGE TO IMAGE Points in the plane move $\qquad$ distances, depending on their distance from the center of rotation. Points farther away from the center of rotation move a distance than those closer to the center of rotation.
Notice how $\overline{A A^{\prime}}, \overline{B B^{\prime}}, \overline{C C^{\prime}}$ are $\qquad$ parallel.

## ORIENTATION

The pre-image has $\qquad$ orientation as its image.

## SPECIAL POINTS

The $\qquad$ is the only point in the plane that is unchanged.

## EQUIVALENT ROTATIONS

## Conterminal angle $=$ initial angle $+360 n$

## SPECIAL ROTATION - ROTATION OF $180^{\circ}$

A rotation of $180^{\circ}$ maps $A$ to $A^{\prime}$ such that:

1. $\mathrm{m} \angle \mathrm{AOA} A^{\prime}=180^{\circ}$ (from definition of rotation)
2. $O A=O A^{\prime}$ (from definition of rotation)
3. Ray $\overrightarrow{O A}$ and Ray $\overrightarrow{O A^{\prime}}$ are opposite rays. $\overleftrightarrow{A O}$ is the same line as $\overleftrightarrow{A A^{\prime}}$
$\qquad$

NOTATION CONSISTENCY

## REFLECTION

A Reflection is recognizable because it will have only ONE item as a subscript... the line of reflection. (Some use a small $r$ for reflection and a capital $R$ for rotation.)
$\boldsymbol{r}_{x}$ axis $\quad$ Reflection over the x axis
$\boldsymbol{r}_{x}$ is probably okay as well
$\boldsymbol{r}_{y \text { axis }} \quad$ Reflection over the y axis
$\boldsymbol{r}_{y}$ is probably okay as well
$\boldsymbol{r}_{x=3} \quad$ Reflection over the $\mathrm{x}=3$ line
$r_{y=x}$ Reflection over the $\mathrm{y}=1 \mathrm{x}$ line
$r_{m} \quad$ Reflection over line $m$
$\boldsymbol{r}_{\overline{A B}} \quad$ Reflection over segment AB
$r_{\overleftrightarrow{A B}} \quad$ Reflection over line $A B$

## TRANSLATION

A translation is recognizable because it will have vector notation.
$T_{\langle-6,4\rangle} \quad$ Translate 6 left and 4 up

## ROTATION

A rotation is recognizable because it will have TWO items in the subscript... a center and a degree.

$$
R_{O, 89^{\circ}}
$$

Rotation about Point O for a positive $89^{\circ}$
When $\mathbf{O}$ is used it is implied that $\mathbf{O}=\mathbf{O r i g i n}$ at $(0,0)$
$R_{P,-134^{\circ}}$
Rotation about Point $P$ for a negative $134^{\circ}$
$\boldsymbol{R}_{(2,3), 42^{\circ}}$
Rotation about location $(2,3)$ for a positive $42^{\circ}$

## DILATION

$D_{O, 3}$ Dilation from point O a scale factor of 3
D
$O, \frac{1}{2}$ Dilation from point O a scale factor of $1 / 2$
$D_{A,-2}$ Dilation from point A a scale factor of -2

## HOW TO WRITE COMPOSITE TRANSFORMATIONS

$$
r_{x a x i s} \circ r_{y=x}(A)
$$

Reflect Point A over the $\mathrm{y}=\mathrm{x}$ line and then reflect that image over the $x$ axis.

$$
r_{y \text { axis }} \circ R_{O, 180^{\circ}}(A)
$$

Rotate A about point $\mathrm{O} 180^{\circ}$ and then reflect that image over the $y$ axis.

$$
r_{x a x i s} \circ T_{\langle-5,3\rangle}(A)
$$

Translate 5 left and 3 up and then reflect that image over the $x$ axis.

NOTICE THAT LIKE COMPOSITE FUNCTIONS WE WORK FROM THE INSIDE OUT. WE WORK RIGHT TO LEFT.....

## Translations Part 2 Terms

## Translations <br>  <br> Key Concept Translation <br> A translation is a transformation that maps all points of a figure the same distance in the same direction. <br> A translation is an isometry. <br> SLIDE <br>  <br> There are three ways to write a translation: <br> $P(x, y) \rightarrow(x+a, y+b)$ or $T_{a, b}$ or $\langle a, b\rangle$ Translations'

Along with translations, Reflect ions are also an_Sometry_. Reflections "flip" an

## Reflections

 image over a line. 个
## Translations Part 2 Terms



〇S Rotations are exactly as you would expect: a transformation that turns an image around a given point. When we are graphing, that point will always be the origin $(0,0)$.
usually

We usually rotate in the same direction that we number the quadrants:
Counter clounmise . If you are asked to rotate clockwise, find the equivalent rotation counterclockwise. (More later...)

| Rule |  | Abbreviation |
| :---: | :---: | :---: |
| Rotation of $90^{\circ}$ about the origin $(x, y)$ |  |  |
| Rotation of $18^{\circ}$ about the origin | $R_{0,90^{\circ}}$ | $(-y, x)$ |
| Rotation of $270^{\circ}$ about the origin | $R_{0,180^{\circ}}$ | $(-x,-y)$ |
| Rotation of $360^{\circ}$ about the origin | $R_{0,270^{\circ}}$ | $(y,-x)$ |

## Translations Part 2 Terms

## Quiz Answer Key

Translations

| Algebraic Rule | Shorthand | Vector Notation |
| :---: | :---: | :---: |
| $P(x, y) \rightarrow(x+a, y+b)$ | $T_{a, b}$ | $\langle a, b\rangle$ |

## Reflections



Rotations
Preimage $(x, y)$

| Rule | Abbreviation | Rule |
| :---: | :---: | :---: |
| Rotation of $90^{\circ}$ about the origin | $R_{O, 90^{\circ}}$ | $(-y, x)$ |
| Rotation of $180^{\circ}$ about the origin | $R_{0,180^{\circ}}$ | $(-x,-y)$ |
| Rotation of $270^{\circ}$ about the origin | $R_{0,270^{\circ}}$ | $(y,-x)$ |
| Rotation of $360^{\circ}$ about the origin | $R_{0,360^{\circ}}$ | $(x, y)$ |

## Translations Part 2 Terms

## Quiz T.A

Translations

| Algebraic Rule | Shorthand | Vector Notation |
| :--- | :--- | :--- |
| $P(x, y) \rightarrow$ |  |  |

## Reflections



Rotations
Preimage $(x, y)$

| Rule | Abbreviation | Rule |
| :---: | :---: | :---: |
| Rotation of $90^{\circ}$ about the origin | $R_{O, 90^{\circ}}$ |  |
| Rotation of $180^{\circ}$ about the origin | $R_{0,180^{\circ}}$ |  |
| Rotation of $270^{\circ}$ about the origin | $R_{0,270^{\circ}}$ |  |
| Rotation of $360^{\circ}$ about the origin | $R_{0,360^{\circ}}$ |  |

## Translations Part 2 Terms

## Quiz T.B

Translations

| Algebraic Rule | Shorthand | Vector Notation |
| :--- | :--- | :--- |
| $P(x, y) \rightarrow$ |  |  |

## Reflections



Rotations
Preimage $(x, y)$

| Rule | Abbreviation | Rule |
| :---: | :---: | :---: |
| Rotation of $90^{\circ}$ about the origin |  |  |
| Rotation of $180^{\circ}$ about the origin |  |  |
| Rotation of $270^{\circ}$ about the origin |  |  |
| Rotation of $360^{\circ}$ about the origin |  |  |

## Translations Part 2 Terms

## Quiz T.C

Translations

| Algebraic Rule | Shorthand | Vector Notation |
| :---: | :---: | :---: |
|  |  |  |

## Reflections

## Rotations

| Preimage $(x, y)$ |  |
| :--- | :---: |
| Abbreviation | Rule |
|  |  |
|  |  |
|  |  |

$\qquad$

## Transformations

A transformation is when an image is changed in some way. The change could be a change is size, shape, or position. The following images have been transformed:


Translations, Reflections and Rotations are called $\qquad$ because the image is congruent to the $\qquad$ _.

## Translations

## Key Concept Translation

A translation is a transformation that maps all points of a figure the same distance in the same direction.
A translation is an isometry.


$$
A A^{\prime}=B B^{\prime}=C C^{\prime}
$$

The diagram at the right shows a translation in the coordinate plane. Each point of the black square moves 4 units right and 2 units down. Using variables, you can say that each $(x, y)$ pair in the original figure is mapped to $\left(x^{\prime}, y^{\prime}\right)$, where $x^{\prime}=x+4$ and $y^{\prime}=y-2$. You can use arrow notation to write the following translation rule.


## Example 1:

## Finding the Image of a Translation

What are the images of the vertices of $\triangle P Q R$ for the translation $(x, y) \rightarrow(x-2, y-5)$ ? Graph the image of $\triangle P Q R$.

Identify the coordinates of each vertex. Use the translation rule to find the coordinates of each vertex of the image.


What does the rule tell you about the direction each point moves?
$x^{\prime}=x-2$ means that each point moves 2 units left. $y^{\prime}=\boldsymbol{y}-5$ means that each point moves 5 units down.

$$
(x, y) \rightarrow(x-2, y-5)
$$

$$
\begin{aligned}
& P(2,1) \rightarrow \\
& Q(3,3) \rightarrow \\
& R(-1,3) \rightarrow
\end{aligned}
$$

To graph the image of $\triangle P Q R$, first graph $P^{\prime}, Q^{\prime}$, and $R^{\prime}$. Then draw $\overline{P^{\prime} Q^{\prime}}, \overline{Q^{\prime} R^{\prime}}$, and $\overline{R^{\prime} P^{\prime}}$.

There are three ways to write a translation:

For example, a translation that moves a point 4 units right and 3 units down can be written as follows:
$P(x, y) \rightarrow$ $\qquad$ or (Algebraic Rule) or

or
(Vector notation)

## Example 2:

## Writing a Rule to Describe a Translation

What is a rule that describes the translation $P Q R S \rightarrow P^{\prime} Q^{\prime} R^{\prime} S^{\prime}$ ?

$\qquad$

## Example 3:

Graph the image of the figure $\mathrm{C}(1,-2), \mathrm{A}(-2,1) \mathrm{T}(-3,-3)$ using
the rule 1 unit left and 2 units up. Then, write the translation rule.


## Example 4:

Write an algebraic rule to describe the transformation:

$$
\begin{aligned}
& C(2,1), O(0,0) L(-5,4), D(-2,1) \\
& \text { to } \\
& C^{\prime}(0,1), O^{\prime}(-2,0) L^{\prime}(-7,4), D^{\prime}(-4,1)
\end{aligned}
$$

## Example 5:

Write an algebraic rule to describe the transformation:

$$
\begin{aligned}
& F(5,-2), R(10,0) E(-5,12), D(0,-3) \\
& \text { to } \\
& F^{\prime}(23,-16), R^{\prime}(28,-14) E^{\prime}(13,-2), D^{\prime}(18,-17)
\end{aligned}
$$



Along with translations, Reflect ions are also an $\qquad$ . Reflections "flip" an image over a line. $\uparrow$

We say
A
Preimage (before)
or $\triangle A B C \overline{\text { is mapped to } \triangle A^{\prime} B^{\prime} C^{\prime}}$

A reflection involves a $\qquad$ of an image, usually over the $x$ or $y$-axis. It may also be flipped over other lines, such as $y=x$ or $x=2$, etc. The best way to graph the image of a reflection is to simply graph the pre-image, measure the distance to the line, and find the image
$\qquad$ on the other side of the line.

Review of commonly used lines:


Use these examples as a reference when reflecting images.

## Notes T. 5 Reflections

## Example 1:

Graph and reflect the preimage $\Delta \mathrm{A}(5,0), \mathrm{B}(3,-1)$ and $\mathrm{C}(4,-3)$ in the $x$-axis.

Did you?:

$\checkmark \quad$ Use a straight-edge? $\checkmark \quad$ Label all points?

## Example 2:

Did you?:

$\checkmark \quad$ Use a straight-edge?
$\checkmark \quad$ Label all points?

| Type of reflection | Abbreviation | Rule |
| :---: | :---: | :---: |
| Reflection in the $x$-axis |  | $(x, y) \rightarrow$ |
| Reflection in the $y$-axis |  | $(x, y) \rightarrow$ |
| Reflection in the $y=x$ |  | $(x, y) \rightarrow$ |
| Reflection in the $y=-x$ |  | $(x, y) \rightarrow$ |

Parallelogram $A(-2,4), B(-3,2), C(1,3), D(2,5)$ is reflected over the line $y=-x$. Graph the preimage and the image and label the coordinates.


Example 4:
Reflect the triangle $\mathrm{A}(-6,2), \mathrm{B}(-5,4)$ and $\mathrm{C}(-4,3)$ in the line $x=-3$.


Example 5:
Find the coordinates of the following figure after a reflection in the line $y=x$.
$F(5,-2), R(10,0) E(-5,12), D(0,-3)$


Rotations are exactly as you would expect: a transformation that turns an image around a given point. When we are graphing, that point will always be the origin $(0,0)$.

We usually rotate in the same direction that we number the quadrants:
$\qquad$ . If you are asked to rotate clockwise, find the equivalent rotation counterclockwise. (More later...)

$\triangle A B C$ is rotated $90^{\circ}$ about point $B$

$\triangle A B C$ is rotated $90^{\circ}$ about point $A$

$\triangle A B C$ is rotated $90^{\circ}$ about point C

Rules for rotating $\qquad$ about the origin:

| Rule | Abbreviation | Transformation |
| :--- | :--- | :--- |
| Rotation of $90^{\circ}$ about the origin | $R_{90^{\circ}}$ | $(\mathrm{x}, \mathrm{y}) \rightarrow$ |
| Rotation of $180^{\circ}$ about the origin | $R_{180^{\circ}}$ | $(\mathrm{x}, \mathrm{y}) \rightarrow$ |
| Rotation of $270^{\circ}$ about the origin | $R_{270^{\circ}}$ | $(\mathrm{x}, \mathrm{y}) \rightarrow$ |
| Rotation of $360^{\circ}$ about the origin | $R_{360^{\circ}}$ | $(\mathrm{x}, \mathrm{y}) \rightarrow$ |

Please keep in mind:

A rotation of $270^{\circ}$ COUNTERCLOCKWISE is equivalent to a rotation of $\qquad$ ! A rotation of $360^{\circ}$ in either direction maps each preimage onto itself.

## Example 1:

Find the coordinates of $\Delta A(2,1), B(3,-1), C(-4,0)$ after a rotation of $90^{\circ}$ counterclockwise about the origin.

## Notes T. 6 Rotations

## Example 2:

Find the coordinates of $\Delta \mathrm{D}(-2,5), \mathrm{E}(0,4), F(-4,-3)$ after a rotation of $180^{\circ}$ counterclockwise about the origin.

## Example 3:

Find the coordinates of $\Delta G(4,-7), H(-2,4), F(-1,0)$ after a rotation of $90^{\circ}$ clockwise about the origin.

## Example 4:

a. Graph trapezoid TRAP where $T(0,4), R(-2,1), A(-5,1)$, and $P(-5,4)$.
b. Graph T'R'A'P', the image of TRAP after $\boldsymbol{R}_{270^{\circ}}$
c. Graph kite KITE where $\mathrm{K}(-3,-3), \mathrm{I}(-1,-3), \mathrm{T}(-1,-1)$ and $\mathrm{E}(-4,0)$.
d. Graph K'I'T'E', the image of KITE after $R_{90^{\circ}}$.


## Symmetry

An object has $\qquad$ if there is a center point around which the object is rotated a certain number of degrees and the object looks the same.

Examples:


Which of the following letters have rotational symmetry?

Which have reflectional symmetry?
Which have reflectional symmetr?

ABCDEF GHIJK MNOPQ RSTHVW XYZ


Geometry 70

1) Dilate $\triangle F A T$ by scale factor of 2 about origin.

$A(1,0) \rightarrow A^{\prime}$
$T(-1,-2) \rightarrow T^{\prime}$
2) Dilate $\triangle F A T$ by scale factor of $\frac{1}{2}$ about origin.

$T(0,-4) \rightarrow T^{\prime}$
3) 


4)

5)

6)

7) Dilate $m$ about the origin by 3 .

8) Dilate $y=\frac{-1}{2} x+4$ about the origin by $\frac{1}{2}$

9) $D_{0,2}(m)=m^{\prime}$

10) Determine the scale factor and center of dilation.

11) Determine if the dilation is an enlargement or reduction.
Scale factor = 3:2
12) Determine if the dilation is an enlargement or reduction.

$$
D_{0, \frac{6}{11}}(\triangle A B C)=\Delta A^{\prime} B^{\prime} C^{\prime}
$$

## Chapter Transformations Summary

1. Summarize the main idea of the chapter
2. Terms (Include name and definition). Also include key example or picture for each term
3. Theorems and Postulates. Also include key example for each theorem or postulate
4. Key examples of the most unique or most difficult problems from notes, homework or application.

## Transformations

1) Reflect FOXY across line $y=x$.

2) Parallelogram SHAQ is shown. Point E is the midpoint of segment SH. Point $F$ is the midpoint of segment $A Q$


Which transformation carries the parallelogram onto itself?
A) A reflection across line segment SA
B) A reflection across line segment EF
C) A rotation of 180 degrees clockwise about the origin
D) A rotation of 180 degrees clockwise about the center of the parallelogram.
3) Square BERT is transformed to create the image $B^{\prime} E^{\prime} R^{\prime} T^{\prime}$, as shown.


Select all of the transformations that could have been performed.
A) A reflection across the line $y=x$
B) A reflection across the line $y=-2 x$
C) A rotation of 180 degrees clockwise about the origin
D) A reflection across the $x$-axis, and then a reflection across the $y$-axis.
E) A rotation of 270 degrees counterclockwise about the origin, and then a reflection across the x -axis.
4) Joe Momma performs a transformation on a hexagon. The resulting triangle is similar but not congruent to the original triangle. Which transformation did Joe Momma perform on the hexagon?
A) Dilation
B) Reflection
C) Rotation
D) Translation
5) Triangle $A B C$ had vertices of $A(1,1), B(3,6)$ and $C(0,2)$. It is dilated by a scale factor of $1 / 4$ about the origin to create triangle $A^{\prime} B^{\prime} C^{\prime}$. What is the length, in units, of side $\overline{B^{\prime} C^{\prime}}$ ?
6) Complete the statement to explain how it can be shown that two circles are similar.

Circle M can be mapped onto circle N by a reflection
across $\qquad$ and a dilation
about the center of circle M by a scale factor of

7) A translation is applied to $\triangle D O G$ to create $\triangle D^{\prime} O^{\prime} G^{\prime}$.


Let the statement $(x, y) \rightarrow(a, b)$ describe the translation. Create equations for $a$ in terms of $x$ and for $b$ in terms of $y$ that could be used to describe the translation.
$a=$ $\qquad$
$b=$ $\qquad$
8) Triangle HEN is shown.


Triangle $H^{\prime} E^{\prime} N^{\prime}$ is created by dilating triangle HEN by a scale factor of 3 . What is the length of $\overline{H^{\prime} E^{\prime}}$ ?
9) A figure is fully contained in Quadrant IV. The figure is transformed as shown.

- A reflection over the $y$-axis
- A reflection over the line $y=x$
- A $90^{\circ}$ clockwise rotation about the origin.

In which quadrant does the resulting image lie?
A) Quadrant I
B) Quadrant II
C) Quadrant III
D) Quadrant IV
10) Rectangle PQRS is shown in the coordinate plane. Points M and N are midpoints of their respective sides.


Select all of the transformations that map the rectangle onto itself.
A) A $90^{\circ}$ clockwise rotation around the center of the rectangle
B) A $180^{\circ}$ clockwise rotation around the center of the rectangle
C) A reflection across $\overline{P R}$
D) A reflection across $\overline{N M}$
E) A reflection across $\overline{Q S}$
11) Triangle $A B C$ is reflected across the line $y=1 / 2 x$ to form triangle RST. Select all of the true statements.
A) $\overline{A B}=\overline{R S}$ (I know this notation is wrong, but some moron used this wrong notation on the state test.)
B) $\overline{A B}=2 \cdot \overline{R S}$ ( 1 know this notation is wrong, but some moron used this wrong notation on the state test.)
C) $\triangle A B C \sim \triangle R S T$
D) $\triangle A B C \cong \triangle R S T$
E) $m \angle B A C=m \angle S R T$
F) $m \angle B A C=2 \cdot m \angle S R T$
12) Triangle BAL is reflected across the line $y=x$. Draw the resulting triangle.

13) All corresponding sides and angles of $\triangle R S T$ and $\triangle D E F$ are congruent.
Select all of the statements that must be true.
A) There is a reflection that maps $\overline{R S}$ to $\overline{D E}$
B) There is a dilation that maps $\triangle R S T$ to $\triangle D E F$
C) There is a translation followed by a rotation that maps $\overline{R T}$ to $\overline{D F}$
D) There is a sequence of transformations that maps $\triangle R S T$ to $\triangle D E F$
E) There is not necessarily a sequence of rigid motions that maps $\triangle R S T$ to $\triangle D E F$
14) The coordinate plane shows $\Delta F G H$ and $\Delta F^{\prime \prime} G^{\prime \prime} H^{\prime \prime}$


Which sequence of transformations can be used to show that $\Delta F G H \sim \Delta F^{\prime \prime} G^{\prime \prime} H^{\prime \prime}$ ?
A) A dilation about the origin with a scale factor of 2 , followed by a $180^{\circ}$ clockwise rotation about the origin.
B) A dilation about the origin with a scale factor of 2, followed by a reflection over the line $y=x$
C) A translation 5 units up and 4 units left, followed by a dilation with a scale factor of $1 / 2$ about point $\mathrm{F}^{\prime \prime}$
D) A $180^{\circ}$ clockwise rotation about the origin, followed by a dilation with a scale factor of $1 / 2$ about $\mathrm{F}^{\prime \prime}$
15) Two triangles are shown.

Which sequence of transformations could be performed on $\triangle E F G$ to show that it is similar to $\triangle J K L$ ?
A) Rotate $\triangle E F G 90^{\circ}$ clockwise about the origin, and then dilate it by a scale factor of $1 / 2$ with a center of dilation at point $\mathrm{F}^{\prime}$
B) Rotate $\triangle E F G 180^{\circ}$ clockwise about point E , and then dilate it by a scale factor of 2 with a center of dilation at point $E^{\prime}$
C) Translate $\triangle E F G 1$ unit up, then reflect it across the $x$-axis, and then dilate it by a factor of $1 / 2$ with a center of dilation at point $E^{\prime \prime}$
D) Reflect $\triangle E F G$ across the $x$-axis, then reflect it across the line $y=x$, and then dilate it by a scale factor of 2 with a center of dilation at point $F^{\prime \prime}$

16) A triangle is shown on the coordinate grid. Draw the triangle after a transformation following the rule $(x, y) \rightarrow(x+5, y-1)$

17) Triangle $A B C$ is dilated with a scale factor of $k$ and a center of dilation at the origin to obtain triangle $A^{\prime} B^{\prime} C^{\prime}$.


What is the scale factor?
18) A pentagon is rotated about its center.

Select all of the angles of rotation that will map the pentagon onto itself.
A) 72 degrees
B) 144 degrees
C) 180 degrees
D) 216 degrees
E) 270 degrees
F) 315 degrees
19) Circle $J$ is located in the first quadrant with center ( $m, n$ ) and radius $k$. Felipe transforms Circle $J$ to prove that it is similar to any circle centered at the origin with radius g .

Which sequence of transformations did Felipe use?
A) Translate Circle J by $(x-m, y-n)$ and dilate by a factor of $\frac{g}{k}$
B) Translate Circle J by $(x-m, y-n)$ and dilate by a factor of $\frac{k}{g}$
C) Translate Circle J by $(x+m, y+n)$ and dilate by a factor of $\frac{g}{k}$
D) Translate Circle J by $(x+m, y+n)$ and dilate by a factor of $\frac{k}{g}$

