Right Triangles – Pythagorean Theorem
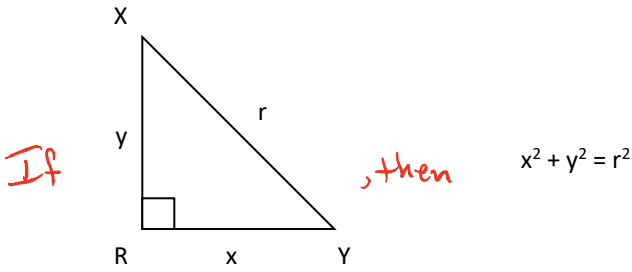
Pythagorean Theorem: In a right triangle, the sum of the squares of the measures of the legs is equals the square of the measure of the hypotenuse.

If

X, r, y, R, x, Y (right triangle diagram)

, then $x^2 + y^2 = r^2$

x and y are always the legs and r is always the hypotenuse.

5 – 12 – 13
9 – 40 – 41

Pythagorean Triple: Three whole numbers that satisfy the Pythagorean Theorem. The smallest Pythagorean Triple is the 3 – 4 – 5 triangle.
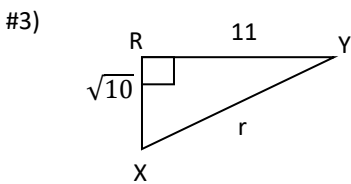
3-4-5
6-8-10
9-12-15
etc

8 – 15 – 17

7-24-25   12-35-37
20-21-29

Use the Pythagorean Theorem to find the missing measure. Give exact answers and rounded answers (if needed) to one decimal place.

#1)

X, 10, (5), 8, y, (4), R, 6, (3), Y
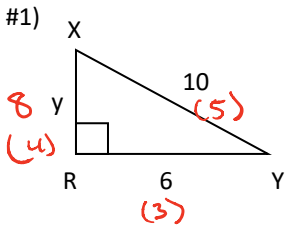
$y = 8$

Pythagorean triple

#2)

X, r, 8, Y, 4, R
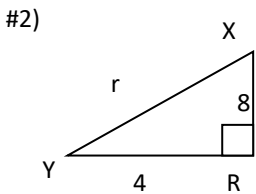
$x^2 + y^2 = r^2$
$(4)^2 + (8)^2 = r^2$
$16 + 36 = r^2$
$52 = r^2$
$\pm\sqrt{52} = r$
$\sqrt{4 \cdot 13} = r$
$2\sqrt{13} = r$
$7.2 \approx r$

#3)

R, 11, Y, $\sqrt{10}$, r, X

$x^2 + y^2 = r^2$
$(11)^2 + (\sqrt{10})^2 = r^2$
$121 + 10 = r^2$
$131 = r^2$
$\pm\sqrt{131} = r$
$\sqrt{131} = r$
$11.4 \approx r$

The converse to the Pythagorean Theorem: If the sum of the squares of the measures of two sides of a triangle equals the square of the measure of the longest side, then the triangle is a right triangle.

If C, r, y, A, x, T (triangle diagram) $x^2 + y^2 = r^2$, then $\triangle CAT$ is ⊿
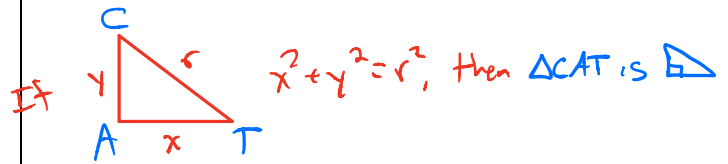
Determine if the following measures can form a right triangle.

#4)   3, 4, 5   Pythagorean Triple

Yes, ⊿

#5)   12, 20, 16

$x^2 + y^2 = r^2$
$(12)^2 + (16)^2 = (20)^2$
$144 + 256 = 400$
$400 = 400$

Yes ⊿

#6)   39, 34, 18

$x^2 + y^2 = r^2$
$(18)^2 + (34)^2 = (39)^2$
$324 + 1156 = 1521$
$1480 \neq 1521$

Not a ⊿

#7)   3.87, 4.47, 5.91

$x^2 + y^2 = r^2$
$(3.87)^2 + (4.47)^2 = (5.91)^2$
$14.9769 + 19.9809 = 34.9281$
$34.9578 \neq 34.9281$

Not a ⊿

#8) In a right triangle, the measures of the legs are 8 and x + 7, and the measure of the hypotenuse is x + 10. Find the value of x.

$$x^2 + y^2 = r^2$$

$$(8)^2 + (x+7)^2 = (x+10)^2$$

$$64 + x^2 + 14x + 49 = x^2 + 20x + 100$$

$$x^2 + 14x + 113 = x^2 + 20x + 100$$

$$-6x + 113 = 100$$

$$-6x = -13$$

$$x = \frac{13}{6}$$

#9) The diagonals of a rhombus measure 30 cm and 16 cm. Use the properties of a rhombus and the Pythagorean Theorem to find the perimeter of the rhombus.
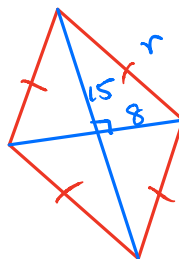
$$x^2 + y^2 = r^2$$

$$(8)^2 + (15)^2 = r^2$$

$$64 + 225 = r^2$$

$$289 = r^2$$

$$\pm 17 = r$$

$$17 = r$$

$$Perimeter = 4r$$

$$= 4(17)$$

$$Perimeter = 68$$