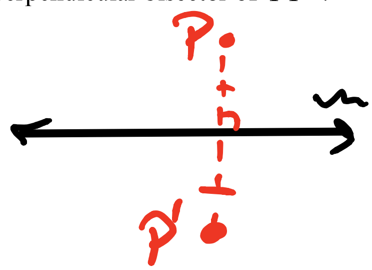
Transformations – Isometric Transformations

G.CO.A.4 Notes Section T.3 Name________________

REFLECTION DEFINITION

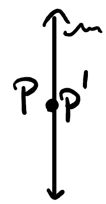
A **reflection** in a line m is a isometric transformation that maps every point P in the plane to a point P', so that the following properties are true:

1. If point P is **NOT** on line m, then line m is the perpendicular bisector of $\overline{PP'}$.



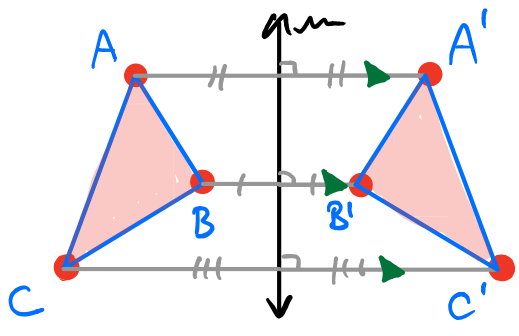
2. If point P is **ON** line m, then P = P'

$r_m(P) = P'$



The line of reflection is the ⊥ bisector of the segment joining every point and its image.

$$r_m(\Delta ABC) = \Delta A'B'C'$$



CHARACTERISTICS

DISTANCES FROM PRE-IMAGE TO IMAGE Points in the plane move different distances, depending on their distance from the line of reflection. Points farther away from the line of reflection move a larger distance than those closer to the line of reflection. Notice how $\overline{AA'} // \overline{BB'} // \overline{CC'}$.

ORIENTATION
The pre-image has a reversed orientation to its image. The reflection creates a mirror image.

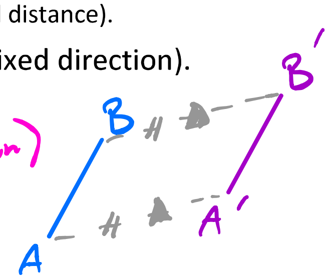
SPECIAL POINTS
The points on the line of reflection do NOT move.

TRANSLATION DEFINITION

A **translation** is an isometric transformation that maps every two points A and B in the plane to points A' and B', so that the following properties are true:

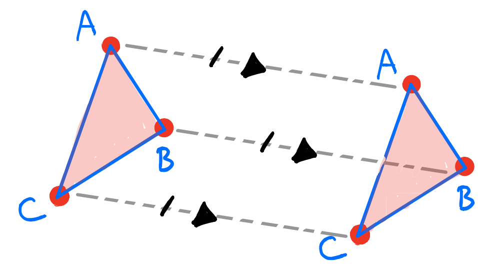
1. AA' = BB' (a fixed distance).
2. $\overline{AA'} \| \overline{BB'}$ (a fixed direction).

vector <x,y>
(left/right, up/down)



$$T_{<x,y>}(\Delta ABC) = \Delta A'B'C'$$

$T_{<5,2>}(\overline{AB})$



CHARACTERISTICS

DISTANCES FROM PRE-IMAGE TO IMAGE
Points in the plane all map the same distance

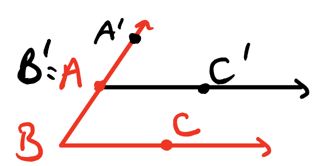
ORIENTATION
The pre-image has same orientation as its image.

SPECIAL POINTS
There are NO special points

SPECIAL TRANSLATION PROPERTY – TRANSLATING AN ANGLE ALONG ONE OF ITS RAYS

A translation of ∠ABC by vector $\overrightarrow{BA}$ maps all points so:

1. ∠ABC ≅ ∠A'B'C' (Isometry)
2. B, A, B' and A' are collinear (translation on angle ray)



Because the two angles are equal and formed on the same ray, then:

$$\overrightarrow{BC} \,\|\, \overrightarrow{B'C'}$$

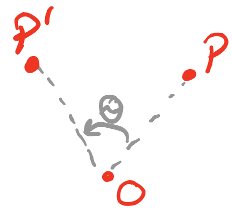
All segments that are translated are parallel to each other.

ROTATION DEFINITION

A **rotation** about a Point O through ϴ degrees is an isometric transformation that maps every point P in the plane to a point P', so that the following properties are true:

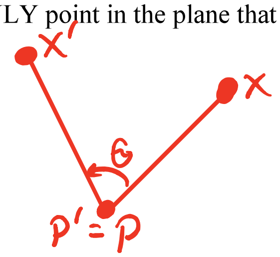
1. If point P is **NOT** point O, then OP = OP' and m∠POP' = ϴ°.

$R_{O,\theta}(P) = P'$



2. If point P **IS** the point of rotation, then P = P'. The center of rotation is the ONLY point in the plane that is unaffected by a rotation.

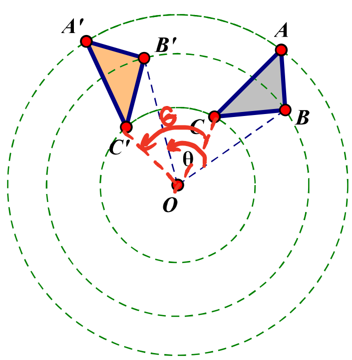
$R_{P,\theta}(\overline{PX}) = \overline{P'X'}$



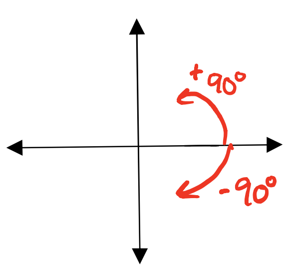
A **rotation** is an isometric transformation that turns a figure about a fixed point called the center of rotation (notation $R_{\text{center, degree}}$).

An object and its rotation are the same shape and size, but the figures may be turned in different directions.

$$R_{O,\theta}(\Delta ABC) = \Delta A'B'C'$$



ROTATION DIRECTION



CHARACTERISTICS

DISTANCES FROM PRE-IMAGE TO IMAGE Points in the plane move ____different____ distances, depending on their distance from the center of rotation. Points farther away from the center of rotation move a distance than those closer to the center of rotation.

Notice how $\overline{AA'}$, $\overline{BB'}$, $\overline{CC'}$ are ____NOT____ parallel.

ORIENTATION

The pre-image has ____Same____ orientation as its image.

SPECIAL POINTS

The ____Center of rotation____ is the only point in the plane that is unchanged.

EQUIVALENT ROTATIONS

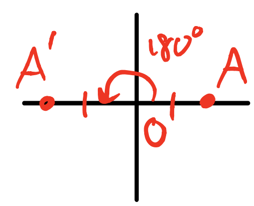
Conterminal angle = initial angle + 360n

SPECIAL ROTATION – ROTATION OF 180°

A rotation of 180° maps A to A' such that:

1. m∠AOA' = 180° (from definition of rotation)
2. OA = OA' (from definition of rotation)
3. Ray $\overrightarrow{OA}$ and Ray $\overrightarrow{OA'}$ are opposite rays.

$\overleftrightarrow{AO}$ is the same line as $\overleftrightarrow{AA'}$



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NOTATION CONSISTENCY

REFLECTION

A Reflection is recognizable because it will have only ONE item as a subscript… the line of reflection. (Some use a small r for reflection and a capital R for rotation.)

$r_{x\ axis}$ Reflection over the x axis

r_x is probably okay as well

$r_{y\ axis}$ Reflection over the y axis

r_y is probably okay as well

$r_{x=3}$ Reflection over the x = 3 line

$r_{y=x}$ Reflection over the y = 1x line

r_m Reflection over line m

$r_{\overline{AB}}$ Reflection over segment AB

$r_{\overleftrightarrow{AB}}$ Reflection over line AB

TRANSLATION

A translation is recognizable because it will have vector notation.

$T_{\langle -6,4 \rangle}$ Translate 6 left and 4 up

ROTATION

A rotation is recognizable because it will have TWO items in the subscript… a center and a degree.

$R_{O,89^\circ}$

Rotation about Point O for a positive 89°

When O is used it is implied that O = Origin at (0, 0)

$R_{P,-134^\circ}$

Rotation about Point P for a negative 134°

$R_{(2,3),42^\circ}$

Rotation about location (2,3) for a positive 42°

DILATION

$D_{O,3}$ Dilation from point O a scale factor of 3

$D_{O,\frac{1}{2}}$ Dilation from point O a scale factor of 1/2

$D_{A,-2}$ Dilation from point A a scale factor of -2

HOW TO WRITE COMPOSITE TRANSFORMATIONS

$r_{x\ axis} \circ r_{y=x}(A)$

Reflect Point A over the y = x line **and then** reflect that image over the x axis.

$r_{y\ axis} \circ R_{O,180^\circ}(A)$

Rotate A about point O 180° **and then** reflect that image over the y axis.

$r_{x\ axis} \circ T_{\langle -5,3 \rangle}(A)$

Translate 5 left and 3 up **and then** reflect that image over the x axis.

NOTICE THAT LIKE COMPOSITE FUNCTIONS WE WORK FROM THE INSIDE OUT. WE WORK RIGHT TO LEFT…..