More Trig - Law of Cosines

Law of Cosines: Let $\triangle A B C$ be any triangle with $a, b$, and $c$ representing the measures of sides opposite angles with measures $A, B$, and $C$ respectively. Then, the following equations hold true.


The law of cosines can be used to solve a triangle in the following cases.

1. To find the measure of the third side of any triangle if the measures of the two sides and the included angle are given.
2. To find the measure of an angle of a triangle if the measures of the three sides are given. (If you are given SSS, YOU CANNOT USE SINES TO FIND THE LARGEST ANGLE.)


For the following examples, round the sides to the nearest tenth and the angles to the nearest whole number.
$\# 1)$ In $\triangle A B C$ if $a=20, c=24$, and $m \angle B=47^{\circ}$, find $b$.


$$
\begin{aligned}
& b^{2}=a^{2}+c^{2}-2 a c \cdot \cos (m \angle B) \\
& b^{2}=(20)^{2}+(24)^{2}-2(20)(24) \cos \left(47^{\circ}\right) \\
& b^{2}=400+576-960 \cos \left(47^{\circ}\right) \\
& b^{2}=976-960 \cos \left(47^{\circ}\right) \\
& b= \pm \sqrt{976-960 \cos \left(47^{\circ}\right)} \\
& b \approx 17.9
\end{aligned}
$$

\#2) In $\triangle A B C$ if $a=5, b=6$, and $c=7$, find $m \angle C$.


$$
\begin{gathered}
c^{2}=a^{2}+b^{2}-2 a b \cos (m \angle c) \\
(7)^{2}=(5)^{2}+(6)^{2}-2(5)(6) \cos (m \angle C) \\
4 a=25+36-60 \cos (m \angle C) \\
49=61-60 \cos (m \angle C) \\
-12=-60 \cos (m \angle C) \\
\frac{12}{60}=\cos (m \angle C) \\
\cos ^{-1}\left(\frac{12}{60}\right)=m \angle C \\
78^{\circ} \approx m \angle C
\end{gathered}
$$



$$
\begin{aligned}
& g^{2}=r^{2}+d^{2}-2 r d \cos (m \angle G) \\
& g^{2}=(100)^{2}+(20)^{2}-2(100)(20) \cos \left(30^{\circ}\right) \\
& g^{2}=10,000+400-4000 \cos \left(30^{\circ}\right) \\
& g^{2}=10,400-4000 \cos 30^{\circ} \\
& g= \pm \sqrt{10,400-4000 \cos 30^{\circ}} \\
& g \approx 83 \text { inches }
\end{aligned}
$$

Rickito and Danny are about 83 inches apart.

